Energy and its Relation to Complexity of the Economy

Granada Seminar 2015
Physics Meets the Social Sciences

Carey W. King
This presentation is a combination of ideas applied to the human economy

Howard T. Odum (systems ecology)
Charles Hall (net energy analysis)
Joseph A. Tainter (anthropology)

Robert E. Ulanowicz (systems ecology, information theory)

Me today (engineer, systems)
Tainter: energy-complexity spiral

• Increased complexity needs resources
  – “… most of the time complexity increases to solve problems.”
  – Societies “… subsequently must produce more energy and other resources to pay for the increased complexity.”


• **Theory:** High gross & net energy resources facilitate increased complexity
• **Research need:** quantitatively test theory for modern society
C. Hall: EROI = energy return on energy invested (measure of net energy)

- How much energy to needed to obtain energy
- Concept is simple

\[
\text{EROI} = \frac{\int \text{Power Output} \, dt}{\int \text{Power input} \, dt}
\]

- I use an interpretation with annual economic data (net power ratio):

\[
\text{NPR}_{\text{econ}} = \frac{\text{GDP}}{\text{$/yr spent by energy and food sectors}}
\]
Ulanowicz: Information theory metrics to quantify distribution of network flows

• **X**, Mutual Constraint
  – Increases with concentration of flows, “efficiency”
  – “number of roles” = \( n = e^X \)

• **Ψ**, Conditional Entropy
  – Increases with more dispersed and uniform flows, “redundancy”
  – “connectivity” = \( m = e^{(\Psi/2)} \)

• **H** = **X** + **Ψ**, Information Entropy
  – **C** = \( T_{..} \times H \) = “capacity”
  – Systems with higher “capacity” can better adapt

\[
X = \sum_{i,j} \frac{T_{ij}}{T_{..}} \ln \left( \frac{T_{ij}T_{..}}{T_iT_j} \right)
\]

\[
\Psi = -\sum_{i,j} \frac{T_{ij}}{T_{..}} \ln \left( \frac{T_{ij}^2}{T_iT_j} \right)
\]

\[
H = -\sum_{i,j} \frac{T_{ij}}{T_{..}} \ln \left( \frac{T_{ij}}{T_{..}} \right)
\]

\( T_{..} \) = total flow in system

Ulanowicz (2009)
Understanding Ulanowicz’s information theory interpretation for networks

• Uniform (fully connected)
  – \( X \) (Mutual Constraint) = 0
  – \( n \) (roles) = 1 (min)
  – \( m \) (connectivity) = 4 (max)
  – ‘roles \times \text{connectivity}’ = 1 \times 4 = 4
  – “redundant” undifferentiated organization

\[
\begin{array}{cccc}
1/4 & 1/4 & 1/4 & 1/4 \\
1/4 & 1/4 & 1/4 & 1/4 \\
1/4 & 1/4 & 1/4 & 1/4 \\
1/4 & 1/4 & 1/4 & 1/4 \\
\end{array}
\]

• Diagonal flows (one path)
  – \( \Psi \) (Conditional Entropy) = 0
  – \( n \) (roles) = 4 (max)
  – \( m \) (connectivity) = 1 (min)
  – ‘roles \times \text{connectivity}’ = 4 \times 1 = 4
  – “efficient” differentiated organization

\[
\begin{array}{cccc}
1 & & & \\
& 1 & & \\
& & 1 & \\
& & & 1 \\
\end{array}
\]
There is a defined ‘role-connectivity’ space for networks

Number of roles \( n = 4 \)

Connectivity \( m = 4 \)

Point of maximum information entropy
Maximum connectivity
Not complex (no distinct parts)
No hierarchy
There is a defined ‘role-connectivity’ space for networks

<table>
<thead>
<tr>
<th>Connectivity (m)</th>
<th>Number of roles (n)</th>
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<tbody>
<tr>
<td>1/4 1/4 1/4 1/4</td>
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</tr>
</tbody>
</table>

Point of maximum information entropy
Maximum connectivity
Not complex (no distinct parts)
No hierarchy

½ maximum information entropy
Maximum effective number of roles
Not resilient (no redundancy)
No hierarchy

Carey W. King | Granada Seminar 2015
June 15, 2015
There is a defined ‘role-connectivity’ space for networks.

- **Number of roles (n):**
  - 4

- **Connectivity (m):**
  - 4

The curve of maximum “role-connectivity” is given by:

\[ m(n) = \frac{4}{n} \]

Key points on the curve:
- **No network:** When \( n = 1 \), \( m(n) = 4 \).
- **Maximum hierarchy:** When \( n = 4 \), \( m(n) = 1 \).
Data: **U.S. Use tables reside at low connectivity and very low number of roles**

- **I-O tables from 1947 to 2012**
  - Bureau of Economic Analysis
  - ~ every 5 years
  - Harmonized to 37 sectors each year to maintain consistency over time

Including: “value added” as input, and GDP as output
Here complexity is the nearness to the point of ‘perfect balance’ @ maximum ‘role-connectivity’ where $n = m = \text{nodes}/2$.
U.S. Use Table complexity versus gross power and net power ratio (NPR),
NPR = GDP/Intermediate food and energy sector transactions

Including value added (input) and GDP (output)
U.S. Use Table complexity versus gross power and net power ratio (NPR),
NPR = GDP/Intermediate food and energy sector transactions

Including value added (input) and GDP (output)
Here hierarchy is the distance from the curve of maximum ‘role-connectivity’

\[ (n \times m)_{\text{max}} = \text{nodes} \]
Hierarchy of U.S. Use I-O transactions has been increasing (shown: intermediate I-O table w/ value added (input) and GDP (output))

NPR = GDP/Intermediate food and energy sector transactions
Hierarchy of U.S. Use I-O transactions has been increasing (shown: intermediate I-O table w/ value added (input) and GDP (output))

NPR = GDP/Intermediate food and energy sector transactions
Conclusions: Generally support that less energy (and energy/capita) → less ‘complexity,’ more ‘hierarchy’

- ‘Complexity’ of U.S. Use matrix decreasing since 1950s
- ‘Hierarchy’ is increasing overall since 1960s
- After 1960s energy/person was no longer increasing exponentially
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Here complexity \( C \) is the nearness to the point of ‘perfect balance’ @ maximum ‘role-connectivity’ where \( n = m = \text{nodes/2} \)

\[
C = \text{nodes} - \left( \left| \frac{\text{nodes}}{2} - n \right| + \left| \frac{\text{nodes}}{2} - m \right| \right)
\]

Complexity normalized to \([0,1]\)

\[
C_{\text{norm}} = \frac{C}{C_{\text{max}}} = \frac{\text{nodes} - \left( \left| \frac{\text{nodes}}{2} - n \right| + \left| \frac{\text{nodes}}{2} - m \right| \right)}{\text{nodes}}
\]
I define hierarchy (Hi) as the distance from the line of maximum "role (n) × connectivity (m)" = $n \times m$

$$Hi = 1 - \frac{n \times m}{(n \times m)_{\text{max}}} = 1 - \frac{n \times m}{\text{number of nodes}}$$

Hierarchy normalized to [0,1]

$$Hi_{\text{norm}} = \frac{Hi}{Hi_{\text{max}}} = \frac{1 - \frac{n \times m}{\text{nodes}}}{1 - \frac{1}{\text{nodes}}} = \frac{\text{nodes} - (n \times m)}{\text{nodes} - 1}$$
A tradeoff of complexity versus hierarchy

Complexity vs. hierarchy (w/ Inputs and Outputs)

Hierarchy vs. Complexity

1947
1958
2012
Hierarchy of U.S. Use I-O transactions has been increasing
(shown: intermediate I-O table only)

NPR = GDP/Intermediate food and energy sector transactions
Conclusions: Generally support that less energy (and energy/capita) → less ‘complexity,’ more ‘hierarchy’

• ‘Complexity’ trends for overall U.S. use matrix
  – Including inputs (wages, profits) and outputs (GDP)
    • Decreasing since 1950s
  – Intermediate matrix only
    • 1947-1997: increasing (food and energy getting cheaper)
    • 1997-2012: decreasing (food and energy getting more expensive)

• ‘Hierarchy’ is increasing overall since 1960s
  – After 1960s energy/person was no longer increasing exponentially
Per Person Graphics
U.S. Use Table complexity versus gross power and net power ratio (NPR) per person

Intercept transactions matrix only

Including value added (input) and GDP (output)
Hierarchy of U.S. Use I-O transactions has been increasing
(shown: intermediate I-O table w/ value added (input) and GDP (output))

NPR = GDP/Intermediate food and energy sector transactions
Hierarchy of U.S. Use I-O transactions has been increasing
(shown: intermediate I-O table only)

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