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Biophysical Modeling of Human and Nature Interactions,

Human and 3-Nature Dynamics Model (HAN3DY)

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Biophysical Modeling of Human and Nature Interactions,

Human and 3-Nature Dynamics Model (HAN3DY)

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Abstract

Biophysical Modeling of Human and Nature Interactions,

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The University of Texas at Austin, 2017.

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The increase in population, and per capita consumption, over the last century have caused an alarming increase in the use of natural resources, causing widespread concern on the sustainability of society. Since all biophysical quantities growing within a finite space would either, i) transition to an equilibrium position, ii) oscillate around an equilibrium state, or iii) overshoot and decline (Meadows, D, et.al, 1972), the Human and three-Nature Dynamics model (HAN3DY) attempts to investigate behavior modes between human and nature through thought experiments. This allows for the understanding of the dynamics of key components (capital, accumulated wealth, population, power, resources) with reference to key parameters (extraction rates, resource levels, etc.). The HAN3DY expands the capabilities of the HANDY model (Motesharrei, S. et.al, 2014) by introducing non-regenerative resources, renewable flow resources, capital, power, and the constant elasticity of substitution (CES) production form to address elasticity of substitution between input parameters in the extraction functions. While HAN3DY reveals that the ultimate cause of societal collapse is the high extraction rate of resources, it also reveals that non-regenerative resources help increase the growth rate of the population, while renewable flow resource acts as a complement to other sources of nature by increasing the carrying capacity of population.

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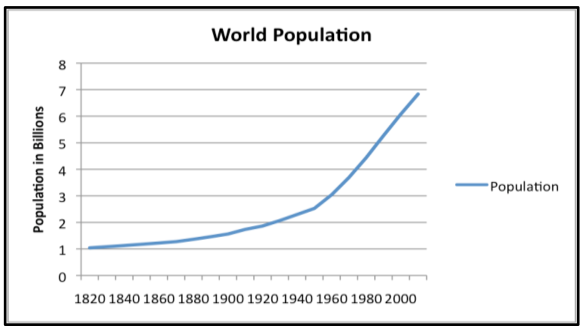
Chapter 1. Introduction and Background

1.1 Introduction

We live on a planet with both finite and renewable natural resources, and over the last century increases in population and per capita consumption have caused an alarming increase in the use of these resources (Tverberg, 2013). The impact on the planet due to these trends can be estimated as the multiplication of the population by per capita ecological foot print (proxy for per capita consumption).



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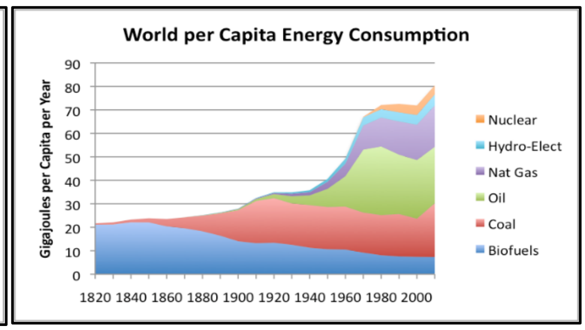

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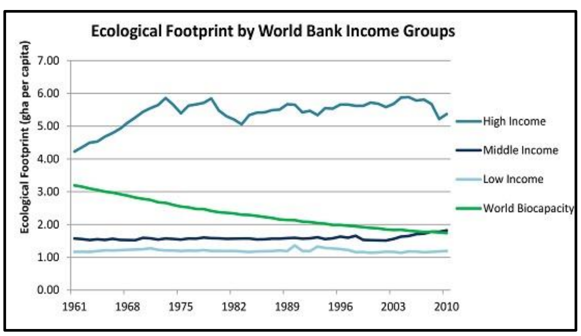

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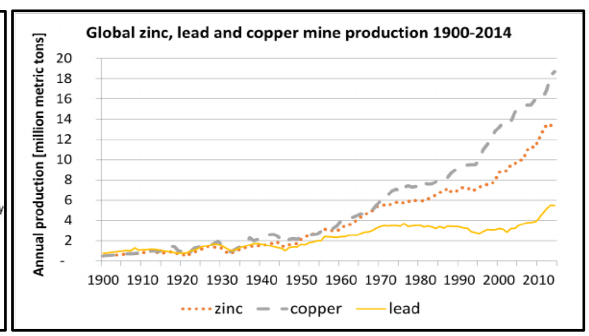

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Hence, there is widespread concern on the sustainability of the society as a whole (Cumming, 2016). A main concern is whether there is a limit to the maximum population size that the environment can sustain indefinitely (maximum carrying capacity) (Cumming, 2016). Additionally, the possibility of the population going over the maximum carrying capacity (overshoot), and whether this could eventually cause an abrupt societal failure (collapse). It is believed that global ecological constraints are having a significant influence on global development due to loss of environmental and ecosystem services (Medows, et.al, 2004). However, aside from ecological feedbacks (which are the effects that change in one part of the ecosystem has on another, and on how this effect feeds back to effect the source), there are also biophysical and economic constraints of capital, labor, and power inputs that might limit the extraction of resources as they are further extracted and depleted.

A system can overshoot (going temporarily beyond sustainable steady state conditions) one or more states or properties. Many systems have negative feedback loops that help bring the system back to a limiting value or carrying capacity either before or after overshoot. However, when there is an exponential growth in some properties (e.g., population, resource usage in society) combined with slow or time delayed negative feedbacks, results could lead to catastrophic events, such as abrupt societal collapses. For these reasons, a thorough understanding on the dynamics of the system is vital since appropriate action could help manage or prevent devastating overshoots.

In order to illustrate this dilemma, the following French riddle is useful. This riddle alludes to the exponential growth and the suddenness with which it approaches its maximum limit. "*If you place a lily pad in an empty pond and it divides to become two lily pads the second day, four lily pads the third day and eight lily pads the fourth day and you know that the lily pond will be completely filled with lily pads on the 30th day choking off all other forms of life, on which day will the pond be half full*?" The answer, is the 29th day, because it is at the end of that day there is only one day to stop the lily pads engulfing the whole pond (Brown, 1978).

The dual stemmed question that is asked is, in which "day" are we now, and is it possible for human civilization to collapse or "choke off" other forms of life that will ultimately impact ourselves? Although there is continuous development in society, through technology and economic specialization to support the growing population, it must be noted that to date, practically all development is sustained through the increased throughput of material and energy.

Although historic events of collapse in civilizations are rare, there have been some instances of strong independent civilizations collapsing such as Mesopotamia (3500BC-1100BC), Roman Empire (753BC-476AD), Greco-Roman Civilization (332BC-339AD), and the Mayan Civilization (1800BC- 800AD). Collapses of these civilizations were followed by long spans of economic, intellectual, and population decline. Since such advanced, sophisticated and creative civilizations have been fragile and impermanent in the past, modern civilization, although larger in context, is also susceptible to collapse (Motesharrei, et.al, 2014).

Human and 3-Nature Dynamics (HAN3DY) model is an advancement of a simpler model HANDY (Human And Nature Dynamics) (Motessharei et al., 2014). The HANDY is a simple biophysical model based on the predator-prey model, and describes the dynamics between the two classes of population (elites and commoners), regenerative nature, and wealth. In HANDY the commoners extract nature in generating wealth, which can be accumulated, which is then consumed by both elites and commoners. This thesis describes additional components to the HANDY model which are (i) the modeling of two additional types of natural resources (non-regenerative nature, and renewable flow nature), (ii) the inclusion of both "capital" and "power", as necessary inputs to extract natural resources, and (iii) production functions that relate extraction inputs (labor, capital, and power) to resource extraction.

In this thesis, Chapter 1 introduces the concept of biophysical modeling, and the biophysical models considered in the development of the HAN3DY model. Chapters 2,3, and 4, describe the systematic development of the model, which comprises 3 versions of the model. Version1 focuses on the disaggregation of the three kinds of nature. Version2 introduces the concept of capital to the system. Version3 introduces the concept of power as wealth consumption or input per unit time. Chapter 5 presents scenarios based on various thought experiments and outcomes.

1.2. Biophysical Modeling

The biophysical environment is the biotic and abiotic surrounding of an organism or population and its influence on the survival, development and evolution. While a model is a simplified representation of reality, biophysical models are the simulation of a biophysical system using mathematical formulas. These models describe the dynamics of each property with relation to other properties in the system, and thereafter it is used to understand the behavior of each property in relation to others. Biophysical modeling typically ignores financial (cash, debt, etc.), social (behavior), and economic (wages, employment, etc.) aspects.

Since social systems are unpredictable and complex, the model presented here is a simplified mathematical model of the more complex socioeconomic system set up to provide long term conditional, projections on the dynamic behavior modes that allow the understanding of the interdependencies between human and nature interactions.

1.2.1. Predator-Prey Model

The “Predator-Prey”, or Lotka-Volterra model is one simple form of a biophysical model developed by Alfred Lotka (Motessharei et al., 2014). In this model species (predators = wolves, prey = rabbits) are interdependent and governed by the Equations [1] and [2].

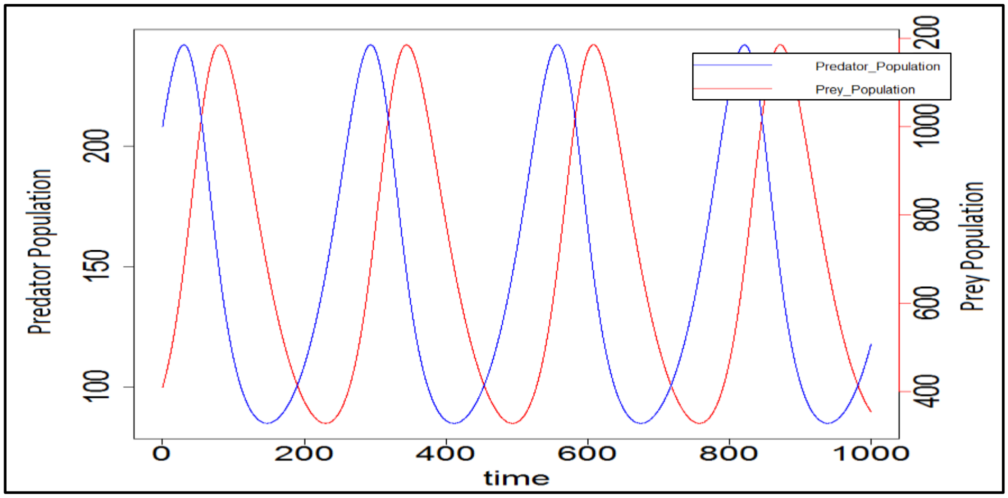
$$\dot{x} = (ay)x - bx \quad [1]$$

$$\dot{y} = cy - (dx)y \quad [2]$$

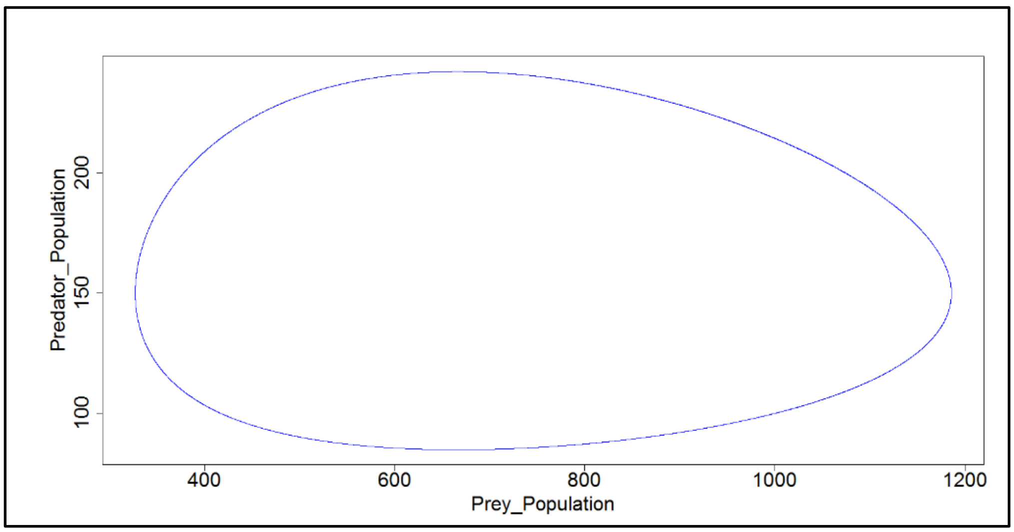
Symbol	Endogenous Variables (EV)/ Parameter (P)
x	Predator Population (EV)
y	Prey Population (EV)
a	Efficiency of predation to growth (P)
b	Predator Death Rate (P)
c	Prey Birth Rate (P)
d	Efficiency of Predation (P)

Table 1.1: Endogenous variables and Parameters describing predator-prey model Equations [1] and [2].

The system is such that growth in the predator population is dependent on a high prey population. Similarly, growth in prey population is dependent on a low predator population. Hence, the two population levels show a cyclic, out of phase variation about the equilibrium values. This model cannot simulate full collapse of the populations due to its structure of interdependency between the two kinds of species. Figure 1.2 is an example simulation of the predator-prey model with equilibrium vales x_e =150 and y_e = 666.66.



(a)



(b)

Figure 1.2: (a) Predator and Prey population levels with time, (b) Predator population vs Prey population level: for parameters, a= $3.0\text{x}10^{-5}$ $\text{time}^{-1}\text{prey}^{-1}$, b= $2.0\text{x}10^{-2}$ time^{-1}, c= $3.0\text{x}10^{-2}$ time^{-1}, d=$2\text{x}10^{-4}$ $\text{time}^{-1}\text{predator}^{-1}$, x(0) = 100, y(0)= 1000.

1.2.2. Human and Nature Dynamics Model (HANDY)

The HANDY model is a biophysical model developed by Safa Motesharrei (Motessharei et al., 2014) that is based on the predator prey model. Its purpose is to understand human population dynamics when considering nature extraction rates and economic inequality among sections in society. In this model, the human population is considered as the predator while natural resources are considered as the prey, which is depleted by the humans. The human population does not necessarily begin to decline upon passing the carrying capacity since it accumulates wealth from the extracted nature resources. The wealth obtained beyond the immediate consumption level of the population is accumulated and stored for future consumption. This model also takes into account the unequal consumption of the accumulated wealth by the introduction of two population classes 'elites' and 'commoners'. Also, HANDY considers that only the commoners are involved in extraction of nature. This model can simulate full societal collapse since there is an intermediate state (wealth) between the predator and the prey.

The governing equations of the HANDY model are:

$$\dot{x}_C = \beta_C x_C - \alpha_C x_C \qquad [3]$$

$$\dot{x}_E = \beta_E x_E - \alpha_E x_E \qquad [4]$$

$$\dot{y} = \gamma y (\lambda - y) - \delta x_C y \qquad [5]$$

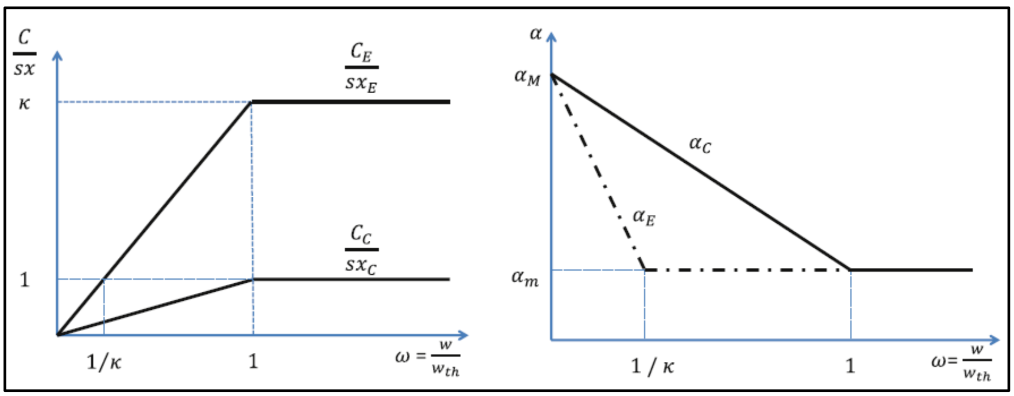
$$\dot{w} = \delta_C x_C y - C_C - C_E \qquad [6]$$

Symbol	Endogenous Variables (EV)/ Parameter (P)	**Units**
$\mathbf{x_C}$	Commoner population (EV)	person
$\mathbf{x_E}$	Elite population (EV)	person
$\mathbf{y}$	Nature (EV)	nature
$\mathbf{w}$	Wealth (EV)	nature
$\boldsymbol{\beta}$	Birth Rate (P)	$time^{-1}$
$\boldsymbol{\alpha}$	Death Rate (EV)	$time^{-1}$
$\boldsymbol{\gamma}$	Regeneration factor (P)	$time^{-1}$
$\boldsymbol{\lambda}$	Nature's Capacity (P)	nature
$\boldsymbol{\delta}$	Extraction rate (P)	$time^{-}1\bullet person^{-1}$
$\mathbf{C_C}$	Consumption by Commoners (EV)	$nature\bullet time^{-1}$
$\mathbf{C_E}$	Consumption by Elites (EV)	$nature\bullet time^{-1}$

Table 1.2: Endogenous variables, parameters and units describing governing Equations of the HANDY model.

The HANDY model is composed of four first order differential equations, Equations [3]-[6]. In these equations, $\mathbf{x_C}$ and $\mathbf{x_E}$ represent the two population classes (commoners and elites) and the growth rates are dependent on the birth rate $\boldsymbol{\beta}$ and death rate $\boldsymbol{\alpha}$ of each (Equations [3]-[4]). HANDY has one kind of nature, **y**, which is regenerative. The rate of change of nature is the positive regenerative component $\boldsymbol{\gamma y\ (\lambda\text{-}y)}$ minus the depletion component $\boldsymbol{\delta x_C y}$. The regenerative component has a regenerative factor (γ**),** and regeneration saturates when nature approaches the capacity limit **(λ)** (Equation [5])**.** The extraction component consists an extraction rate **(δ)** multiplied by the commoner population ($\mathbf{x_C}$), which works in extracting the nature, multiplied by the amount of nature, y. The more nature is depleted, the slower extraction occurs. The extracted nature is converted to wealth (**w**). The accumulated wealth is the difference between the extraction of nature and the consumption by the commoners (**C_c**) and the elites (**C_E**) (Equation [6]).

The model typically consists of a constant birth rate (**β =0.03),** while the death rate and the consumption rates are dependent on the accumulated wealth in the system (which is an economic indicator of the society), and governed as per Figure 1.3.



(a) (b)

Figure 1.3 : (a) Consumption rates and (b) death rates for elites and commoners, as a ratio of $(\frac{w}{w_{th}})$ (Motesharrei, et.al , 2014).

The wealth threshold **w_{th}** (Equation[7]), is a designated threshold value for wealth for the total population below which famine starts, and the death rates vary between **α_m** (healthy death rate) and **α_M** (famine death rate). α_c and α_E, are the death rates of the commoner and elite populations, and as per Equations [8] and [9].

$$w_{th} = \rho x_C + \kappa\rho\, x_E \qquad [7]$$

$$\alpha_C = \alpha_m + \max\left(0, 1- \frac{C_C}{s x_C}\right) (\alpha_M - \alpha_m) \qquad [8]$$

$$\alpha_E = \alpha_m + \max\left(0, 1- \frac{C_E}{s x_E}\right) (\alpha_M - \alpha_m) \qquad [9]$$

Symbol	Endogenous Variables (EV)/ Parameter (P)	Units
w_{th}	Wealth threshold value (EV)	
ρ	Threshold wealth per capita (P)	wealth•person^{-1}
κ	Inequality Factor (P)	-
s	Subsistence Salary per Capita (P)	wealth•person^{-1}•time^{-1}
α_M	Maximum Death rate (P)	time^{-1}
α_m	Minimum Death Rate (P)	time^{-1}

Table 1.3: Endogenous variable, parameters and units describing death rates in HANDY, Equations [7],[8] and [9].

The consumption rates for the commoners and the elites are as Equations [10] and [11], with there being an inequality factor (**κ**), indicating the per capita consumption by the elites as a multiple of per capita consumption of commoners ($\kappa \geq 1$).

$$C_C = min\left(1, \frac{w}{w_{th}}\right) s x_C \qquad [10]$$

$$C_E = min\left(1, \frac{w}{w_{th}}\right) \kappa s x_E \qquad [11]$$

Three sets of scenarios can be simulated in this model, they are

1) Egalitarian Society with no elites $\mathbf{x_E = 0}$

2) Equitable society, $\mathbf{x_E > 0}$, $\boldsymbol{\kappa} = 1$

3) Unequal society, $\mathbf{x_E > 0}$, $\boldsymbol{\kappa} > 1$

Select simulation results from (Motessharei et al., 2014) are shown in Figures 1.4-1.6.

Egalitarian Society

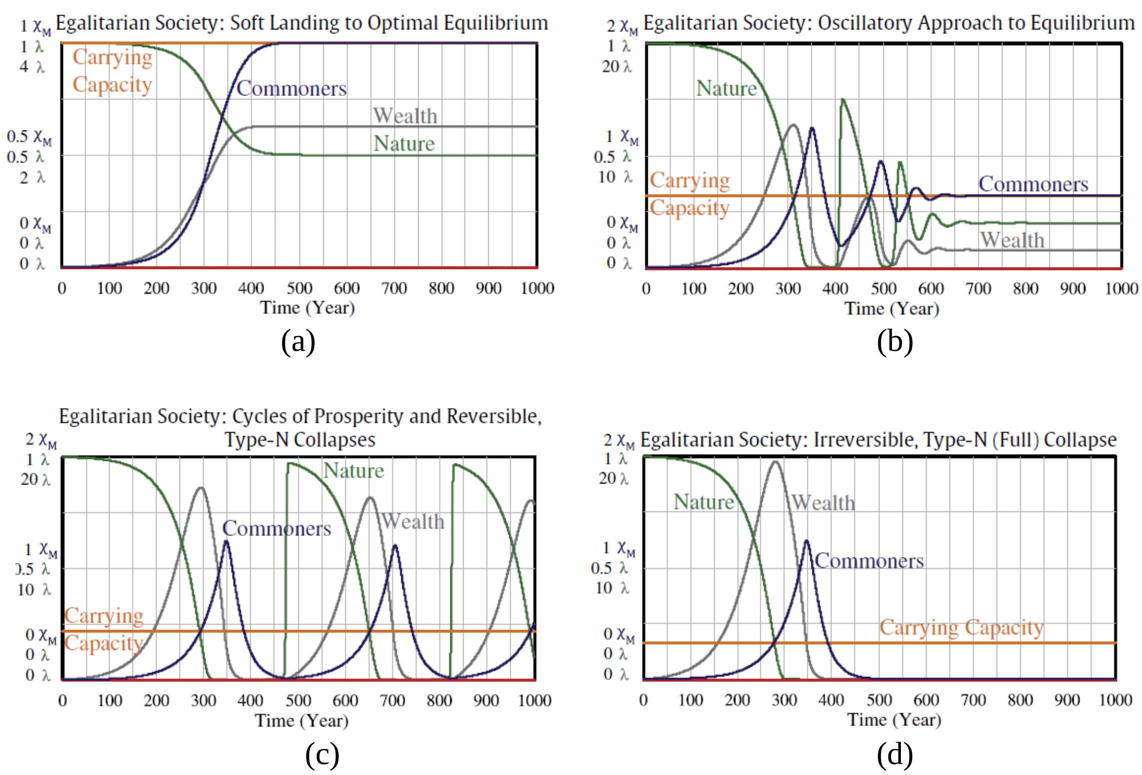


Figure 1.4 : Experimental results of the state variable (wealth, population and nature quantities) for an egalitarian Society as extraction rate, δ, increases: (a) $\delta = \delta^* = 6.67 \times 10^{-6}$ [1/person/year], (b) $\delta = 1.66 \times 10^{-5}$, (c) $\delta = 2.66 \times 10^{-5}$, and (d) $\delta = 3.66 \times 10^{-5}$ (Motesharrei, S. et.al ,2014)

Using the parameter values as per in Appendix 1,Table 1, Figures 1.4(a)-(d) show four simulation results while varying only the extraction factor δ. The extraction factor is increased from its “optimal” value, δ*, which maximizes population to its carrying capacity at an equilibrium steady state with no overshoot. When this optimal extraction factor is increased by 2.5, 4 and 5.5 times in (Figure 1.4 (b) to (d)), it is seen that system moves from a soft landing equilibrium (optimal equilibrium) in Figure 1.4(a) to an eventual irreversible collapse in Figure 1.4(d). The results convey that high extraction rates of nature would lead to system instability and eventual collapse.

Equitable society

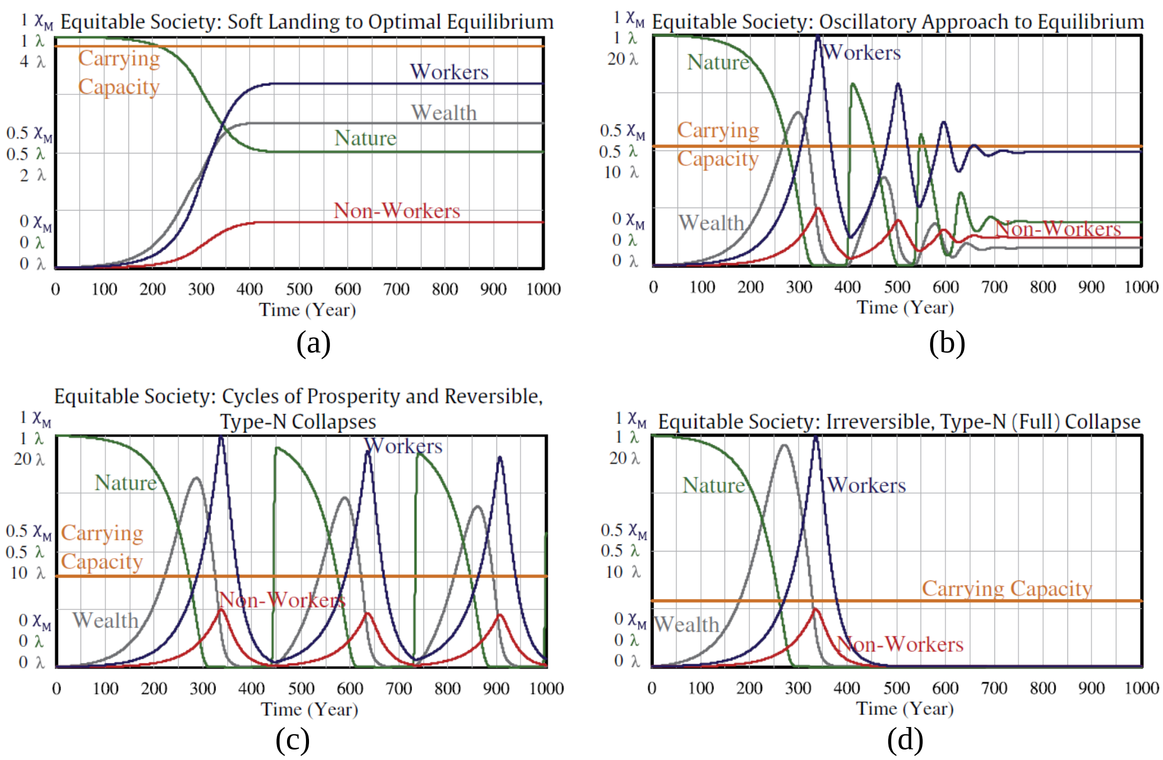
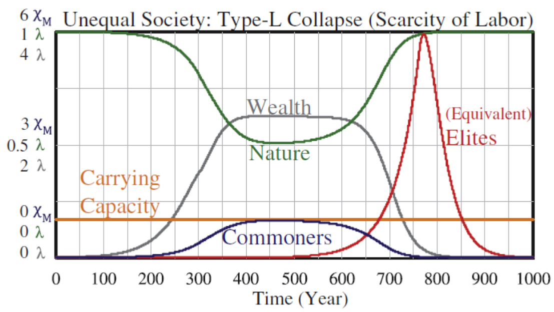


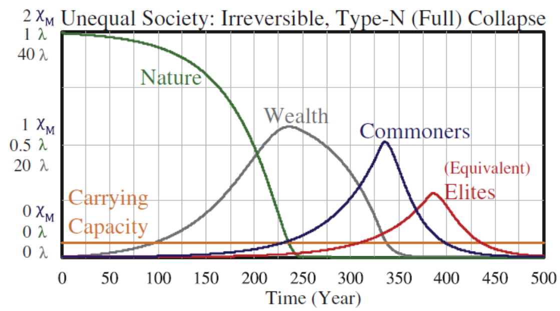
Figure 1.5: Experimental results of the state variable (wealth, population and nature quantities) for the Equitable Society with initial elite population (x_E=10), as extraction rates increase: (a) δ = δ* =8.33x10-6 [1/person/year], (b) δ =2.20x10-5, (c) δ = 3.00x10-5 , and (d) δ = 4.33x10-5 (Motesharrei, S. et.al, 2014).

The results of the equitable society is also similar to the egalitarian society, with the system becoming unstable and causing eventual collapse with the increase of the extraction rate to 2.6, 3.4 and 5 times the optimal maximized extraction rate (8.33×10^{-6}) Figure 1.5 (a)-(d).

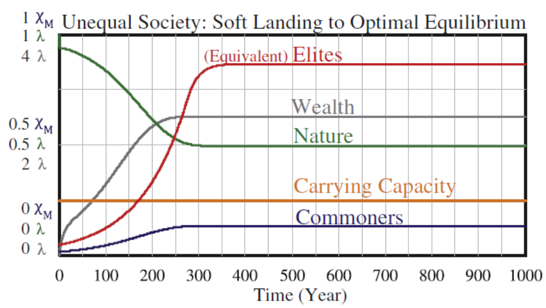
Unequal society



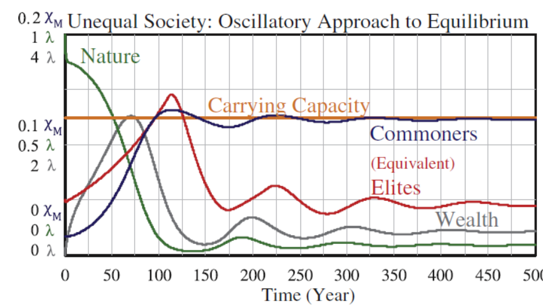
(a) When κ is high (100)



(b) When κ and δ are high



(c) Moderate κ (10)



(d) Moderate κ with high δ

Figure 1.6: Experimental results of the state variable (wealth, population and nature quantities) for the Unequal Society (a) $\delta = 6.67 \times 10^{-6}$ [1/person/year], (b) $\delta = 1 \times 10^{-4}$, (c) $\delta = 6.35 \times 10^{-6}$, and (d) $\delta = 1.35 \times 10^{-5}$.

In the unequal society, it is seen that there are oscillations and eventual collapse from optimal equilibrium conditions, either when the extraction factor is high or when the inequality factor is high. In comparing Figures 1.4(a) and 1.6(a), the only difference is the presence of elites consuming at a high inequality factor. The unequally high consumption rate by the elites reduces the accumulated wealth, causing the increase in the commoner death rate, which is dependent on the accumulated wealth. With the reduction in commoners the wealth generation reduces, yet the death rates of the elites do not increase until $(\frac{w}{w_{th}})$ reduces below $(\frac{1}{\kappa})$ as illustrated in Figure 1.3(b), causing the growth in the elite population. When the $(\frac{w}{w_{th}} < \frac{1}{\kappa})$ the collapse in the elite population is rapid.

The model, brings out the idea that either the over exploitation of natural resources or the strong economic stratification can independently result in an abrupt societal failure.

1.2.3. World3 Model

The Limits to Growth (Meadows et al. 1972) describes one of the earliest applications of computer modeling to global environmental challenges. The authors used the World3 model developed by Professor Jay Forrester of MIT (Forrester 1971). The World3 model is a system dynamics model based on interactions between the population, capital sector, agriculture sector, nonrenewable sector, and the persistent pollution. While the World3 model is calibrated to actual data at the global scale, all natural resources except soil fertility (which is considered to be a regenerative resource aiding agriculture) are lumped into non-regenerative resources. One of the main concluding remarks gained though the study is that a system processing three characteristics, rapid growth, environmental limits and feedback delays, is inherently unstable, and the standard run of the model simulates overshoot of the system dynamics (e.g., total population) around the year 2015 and then eventual decline. The *Limits to Growth* (Meadows et al. 1972) book consists of thirty simulations chosen to promote an understanding of the dynamic properties of the World3 model and to demonstrate the effectiveness of alternative policies. Figure 1.7 is the standard World3 model run assuming no major changes in physical, economic or social relationships that have historically governed the development of the world system. As per Figure 1.7 it is seen that while the non-regenerative resources continue to decline, all other factors (population, food, industrial output, and pollution) peak and then decline.

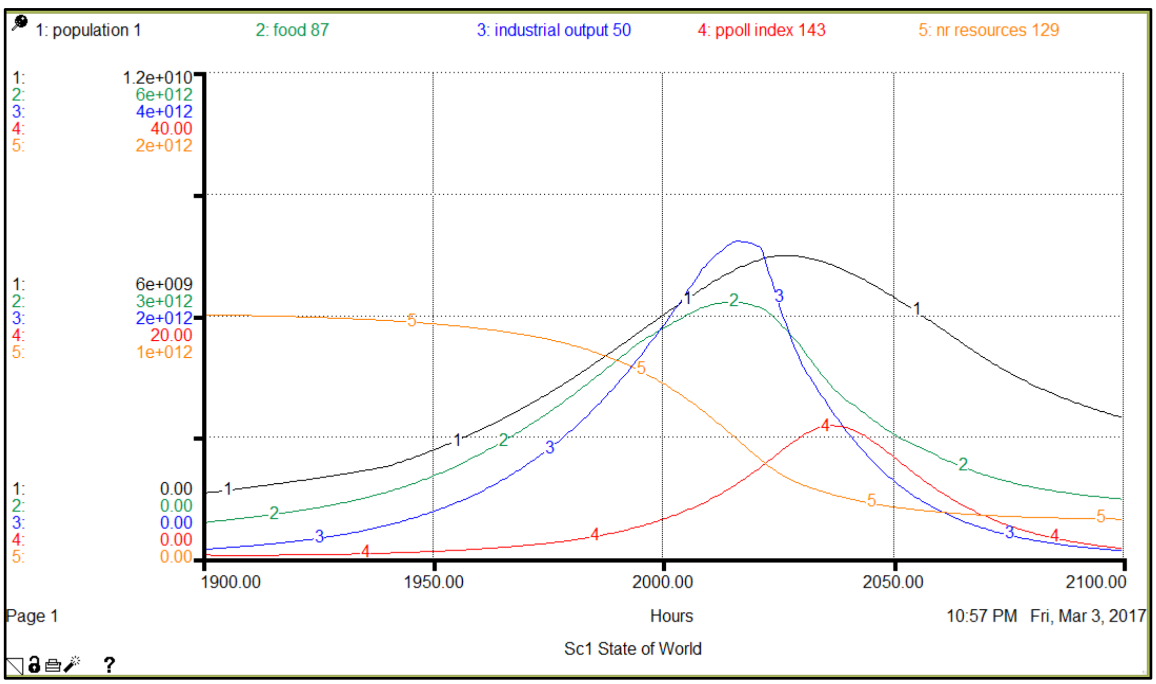


Figure 1.7: State of the World Dynamics as per the World3 Model's "Standard" run (result from the model using the STELLA software).

1.3. Research Question.

The system dynamics models discussed in Section 1.2 describe relationships between population and natural resource extraction. World3 assumes only the use of finite, or non-regenerative, natural resources (e.g., fossil fuels, and minerals), as an input that is required to enable all activities in the economy and HANDY assumes only use of regenerative resources (e.g., a forest). Hence, this thesis explores a model with three types of resources (regenerative, non-regenerative and renewable flows), and their characteristics in understanding the macro scale dynamics between the human and nature parameters. This is called the HAN3DY model, which stands for Human and 3-Nature Dynamics. The goal of this thesis is to develop the modeling framework that can be used to understand the tradeoffs of using the three fundamentally-different resources. The model can be used to provide insight into how different types of natural resources, and the proportion and timing of use of each, relates to the dynamics of population and wealth accumulation, which would then enable the study of an energy transition.

1.4. Methodology

HAN3DY is a simplified mathematical model, tracking human and nature interactions of a complex social system. The method used to track, translate, and analyze, the interactions in the HAN3DY model is through system dynamics. The central concept for system dynamics is to understand how all the objects in a system interact with one another. This methodology uses stocks, flows, feedback loops, table functions and time delays in understanding the nonlinear behavior of complex systems (MIT-System Dynamics in Education Project)..

In developing the model using system dynamics the specific states to be modeled were identified (e.g., population, nature, wealth, capital, and power), and disaggregated. Next a set of equations were created, that are consistent in tracking stocks (nature, wealth, and capital), that gets allocated for various purposes (consumption, accumulation, and investment) over time. Also feedbacks were introduced between parameters, in order to understand their influence upon growth and decline of the various stocks over time. HANDY is the starting point for developing the new model, 'HAN3DY'. In the following chapters the primary HANDY model components have been modified and disaggregated through a number of logical steps in sequence, each dependent on the completion of the one before.

1.4.1. Validation of the Model

Throughout the development of the HAN3DY model, a combination of validation techniques were used in verifying the model (Sargent, 2004). Some of the validation techniques used were:

1. Animation: All output parameters were tracked graphically, through time, which allowed to understand the operational behavior of the model.

2. Comparison to Other Models: The results of the models were continuously compared to the results of the HANDY model.
3. Degenerate Tests: Model was tested by appropriate values of the input and internal parameters. The results of these tests helped verify the model.
4. Extreme Condition Tests: Model was tested for extreme and unlikely conditions, to ensure the results would reflect the testing conditions. For example, the major goal of this thesis is to integrate two new nature resources into the HANDY model. In testing the model developed in this thesis, the model was simulated with at extreme conditions with only one type of nature resource to compare to cases when all three were included.
5. Parameter Variability - Sensitivity Analysis: Single parameter values were varied, to understand impacts on model behavior. For example, regenerative nature extraction rate was varied, while other parameters were kept constant, in understanding the dynamics due to the change in extraction rate (see simulation 2.3.1.(i) and Figure 2.2).
6. Traces: Behavior of different, output parameters were traced, in determining the correctness of the logic used.

Chapter 2: Disaggregation of Nature

2.1. Introduction

This chapter introduces version1 of HAN3DY model. Version1 of HAN3DY model expands the scope of HANDY by creating two additional types of nature: non-regenerative nature (e.g., fossil fuels) and renewable flows (e.g., sunlight, wind). In this section, the disaggregation of the nature component is introduced, with the relevant modifications to the primary HANDY model equations and parameters.

2.2. Model Development

Population consists of two classes' commoners and elites, which are dependent on the birth, and death rates. While the birth rate is set to a constant value the model incorporates economic factors that influence the death rates and thus, the population size. The expanded HAN3DY model keeps the same structure for the population equations as per HANDY.

$$\dot{x}_C = \beta_C x_C - \alpha_C x_C \qquad [3]$$

$$\dot{x}_E = \beta_E x_E - \alpha_E x_E \qquad [4]$$

Nature represents the source of wealth generation to meet the population consumption requirements and enable population growth. As different nature sources have different characteristics based on their regeneration, quantities, and methods of usage, the first major step in development of HAN3DY consists of creating equations for the new nature types to include with the original regenerative nature in HANDY, which is governed by the Equation [5].

$$\dot{y} = \gamma y\,(\lambda - y) - \delta x_c y \qquad [5]$$

Kinds of Nature:

As indicated in Equation [5], the HANDY model only consists one kind of nature, a regenerative stock. Whereas in reality, resources exist in multiple forms namely regenerative stocks (forests, aquifers, soils, etc.), non-regenerative stocks (fossil fuels, mineral deposits, etc.) and renewable flows (wind, solar radiation, etc.). Hence, in order to make the model more relevant to prevailing real world conditions, three nature components with varying characteristics have been introduced.

a) Regenerative Nature

The equation for regenerative stocks is the same as the nature equation in HANDY model, with one exception. Only a fraction of the total commoner population ($\mathbf{x_{c,R}}$) is assumed to work towards extracting regenerative nature, with others working on extracting the other two kinds of nature.

$$\dot{y}_R = \Upsilon \mathbf{y_R}(\boldsymbol{\lambda}_\mathbf{R} - \mathbf{y_R}) - \boldsymbol{\delta}_\mathbf{R}\, \mathbf{x_{c,R}} \mathbf{y_R} \qquad [12]$$

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
$\mathbf{y_R}$	Regenerative nature (EV)	nature
$\boldsymbol{\lambda}_{\mathbf{R}}$	Regenerative nature carrying capacity (P)	nature
$\mathbf{x_{cR}}$	Commoners working on regenerative nature (EV)	person
$\boldsymbol{\delta}_{\mathbf{R}}$	Extraction rate of regenerative nature (P)	person^{-1}•time^{-1}
Υ	Regeneration rate (EV)	nature^{-1}•time^{-1}

Table 2.1: Description and units of endogenous variables and parameters used in Equation [12]

b) Non-Regenerative Nature

The equation structure for non-regenerative resource (NR resource) stock, is similar to the regenerative stock equation with the regenerative component being zero, since there is no regeneration of this kind of resource ($\Upsilon = 0$). Hence, non-regenerative nature would continuously decline from its initial stock, with extraction. Here too, only a fraction of the population **($\mathbf{x_{c,NR}}$)** works towards extracting the nature.

$$\dot{y}_{NR} = -\,\boldsymbol{\delta}_{\mathbf{NR}}\,\mathbf{x_{c,NR}}\boldsymbol{y}_{\mathbf{NR}} \quad \textbf{[13]}$$

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
$\mathbf{y_{NR}}$	Non-regenerative nature (EV)	nature
$\mathbf{x_{c,NR}}$	Commoners working on non-regenerative nature (EV)	person
$\boldsymbol{\delta}_{\mathbf{NR}}$	Extraction rate of non-regenerative nature (P)	person^{-1}•time^{-1}

Table 2.2: Description and units of endogenous variables and parameters used in Equation [13]

c) Renewable Flow Nature

Renewable flow nature has been introduced in order to replicate the use of "flow" natural resources, such as using technology to convert solar irradiation and wind power into electricity. The equation for flow-based nature calculates the amount of land allocated to setting up such generating plants to extract flow-based resources.

$$\dot{y}_{RF} = \boldsymbol{\delta}_{\mathbf{RF}}(\boldsymbol{\lambda}_F - \mathbf{y}_{\mathbf{RF}})\mathbf{x}_{\mathbf{c,RF}} \quad \textbf{[14]}$$

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
y_{RF}	Renewable flow nature extracted (Land) (EV)	land
λ_F	Renewable flow nature carrying capacity (Maximum available land) (P)	land
$x_{c,RF}$	Commoners working on renewable flow nature (EV)	person
δ_{RF}	Extraction rate of renewable flow nature (Land) (P)	$person^{-1}$•$time^{-1}$

Table 2.3: Description and units of endogenous variables and parameters used in Equation [14].

The land area used for extracting flows is initially assumed to be zero, and has a maximum possible capacity of λ_F (maximum available land to harness flow resources). The equation is such that when the land being extracted reaches saturation limits, the rate at which land is extracted declines to zero. Similar to the two previous equations, the rate of land being acquired is dependent on the extraction rate (δ_{RF}) and the amount of the commoner population ($x_{c,RF}$) working on it.

2.2.1 New Parameters for version1 of HAN3DY model

Wealth

Since there are two kinds of resource stock extractions (regenerative and non-regenerative stocks), the wealth accumulated from each kind of resources are as Equations [15] and [16].

$$\dot{w}_R = \delta_R \, x_{c,R} y_R - C_{C,R} - C_{E,R} \qquad [15]$$

$$\dot{w}_{NR} = \delta_{NR} \, x_{c,NR} y_{NR} - C_{C,NR} - C_{E,NR} \qquad [16]$$

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
w_R	Accumulated regenerative wealth (EV)	nature
w_{NR}	Accumulated non-regenerative wealth (EV)	nature
$C_{C,R/NR}$	Consumption of commoners, regenerative/ non-regenerative (EV)	nature•time^{-1}
$C_{E,R/NR}$	Consumption of elites, regenerative/ non-regenerative (EV)	nature•time^{-1}

Table 2.4: Description and units of endogenous variable and parameters used in Equations [15],[16]

Since both regenerative and non-regenerative stocks are physical entities, the extracted resource components from nature is considered as wealth and can be accumulated. The accumulation rate of these kinds of wealth (w_R, w_{NR}) are dependent on the extracted quantities and consumption of it from both the commoners and elites as defined in Equations [15] and [16].

In order to keep the model simple, it is assumed that regenerative and non-regenerative resource stocks are substitutable and the total wealth accumulation equation would be as Equation [17].

$$\dot{w} = \boldsymbol{\delta_R\, x_{c,R} y_R + \delta_{NR}\, x_{c,NR} y_{NR} - C_{C,NR} - C_{E,NR} - C_{E,R} - C_{C,R}} \quad \textbf{[17]}$$

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
w	Total accumulated wealth (EV)	nature
$C_{E,R}$	Regenerative stock consumption by elites (EV)	nature•time^{-1}
$C_{E,NR}$	Non-regenerative stock consumption by elites (EV)	nature•time^{-1}
$C_{C,R}$	Regenerative stock consumption by commoners (EV)	nature•time^{-1}
$C_{C,NR}$	Non-regenerative stock consumption by commoners (EV)	nature•time^{-1}

Table 2.5: Description and units of endogenous variable and parameters used in Equation [17].

Power

In HAN3DY model power, is assumed as the sole output that is produced from renewable flow resources. This renewable flow power is generated from the extracted renewable flow land (y_{RF}) with the use of capital (e.g., solar panels and wind turbines used to harness the renewable flow resource). In calculating the renewable flow power generation, the Equation[18] was used.

Assuming y_{RF} to be the amount of land being used

$$\boldsymbol{P = \int_0^{y_{RF}} p(y_{RF})\, dy_{RF}} \quad \textbf{[18]}$$

Here $p(y_{RF})$ is the power extraction density (power·land^{-1} = nature·time^{-1}·land^{-1}) allocated to extracting renewable flows as a function the amount of land allocated to renewable flows.

When assumed, that $p(y_{RF})$ = constant = **I**, there is a constant power generation per land area for the entire quantity of extracted land, the power being generated is governed by the Equation [19].

$$\boldsymbol{P_{RF} = I y_{RF}} \quad \textbf{[19]}$$

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
P_{RF}	Power capacity from renewable flow resources (EV)	nature•time^{-1}
I	Resource quality (P)	time^{-1}

Table 2.6: Description and units of endogenous variable and parameters used in Equation [19]

<u>Consumption</u>

The total consumption by the population (regenerative stock and non-regenerative stock) are as Equations [20] and [21]. Consumption levels are based on the ratio ($\frac{(\boldsymbol{w_R + w_{NR}})}{\boldsymbol{w_{TH}}}$) which is the accumulated wealth compared to the wealth threshold value. This ratio is used as an economic indicator similar to the HANDY model. Hence, if the total accumulated wealth ($\boldsymbol{w_R + w_{NR}}$) is greater than the total threshold value ($\boldsymbol{w_{TH}}$), the consumption per capita is the subsistence salary ($\mathbf{s}$). If the accumulated total wealth $(\boldsymbol{w_R} + \boldsymbol{w_{NR}})$ is lower than the threshold value, the consumption too declines linearly as per Figure 1.3(a).

$$\mathbf{C_c} = \boldsymbol{min}(\mathbf{1}, (\frac{(\boldsymbol{w_R}+\boldsymbol{w_{NR}})}{\boldsymbol{w_{TH}}}))\mathbf{sx_c} \qquad \textbf{[20]}$$

$$\mathbf{C_E} = \boldsymbol{min}(\mathbf{1}, (\frac{(\boldsymbol{w_R}+\boldsymbol{w_{NR}})}{\boldsymbol{w_{TH}}}))\ \boldsymbol{\kappa}\mathbf{sx_E} \qquad \textbf{[21]}$$

The consumption of regenerative and non-regenerative stocks, are individually tracked as per the Equations [22] - [25], such that accumulated wealth $(\boldsymbol{w_R}, \boldsymbol{w_{NR}})$ can be individually accounted for in Equations [15] and [16].

$$\mathbf{C_{C,R}} = \boldsymbol{min}(\mathbf{1}, (\frac{\boldsymbol{w_R}}{\boldsymbol{w_{TH\_R}}}))\ \mathbf{s}_{\boldsymbol{R}}\mathbf{x}_{\boldsymbol{C}} \qquad \textbf{[22]}$$

$$\mathbf{C_{E,R}} = \boldsymbol{min}(\mathbf{1}, (\frac{\boldsymbol{w_R}}{\boldsymbol{w_{TH\_R}}}))\ \boldsymbol{\kappa}\mathbf{s}_{\boldsymbol{R}}\mathbf{x}_{\boldsymbol{C}} \qquad \textbf{[23]}$$

$$\mathbf{C_{C,NR}} = \boldsymbol{min}(\mathbf{1}, (\frac{\boldsymbol{w_{NR}}}{\boldsymbol{w_{TH\_NR}}}))\ \mathbf{s}_{\boldsymbol{NR}}\mathbf{x}_{\boldsymbol{C}} \qquad \textbf{[24]}$$

$$\mathbf{C_{E,NR}} = \boldsymbol{min}(\mathbf{1}, (\frac{\boldsymbol{w_{NR}}}{\boldsymbol{w_{TH\_NR}}}))\ \boldsymbol{\kappa}\mathbf{s}_{\boldsymbol{NR}}\mathbf{x}_{\boldsymbol{C}} \qquad \textbf{[25]}$$

If $\mathbf{s} = \mathbf{s}_{\boldsymbol{R}} + \mathbf{s}_{\boldsymbol{NR}}$, and $\mathbf{s}_{\boldsymbol{R}} = \mathbf{s}_{\boldsymbol{NR}}$, then $\boldsymbol{w_{TH\_R}} = \boldsymbol{w_{TH\_NR}} = \frac{\boldsymbol{w_{TH}}}{\mathbf{2}}$.

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
C_C	Total commoner consumption (EV)	nature
C_E	Total elite consumption (EV)	nature
$C_{E,R/NR}$	Regenerative and non-regenerative stock consumption by elites (EV)	nature
$C_{C,R/NR}$	Regenerative and non-regenerative stock consumption by commoners (EV)	nature•time^{-1}
w_{TH_R}	Wealth threshold wealth- regenerative stocks (EV)	nature
w_{TH_NR}	Wealth threshold wealth- non-regenerative stocks (EV)	nature
κ	Inequality Factor (P)	-
s_R	Subsistence salary per capita - regenerative stocks (P)	nature•time^{-1}•person^{-1}
s_{NR}	Subsistence salary per capita - non-regenerative stocks (P)	nature

Table 2.7: Description and units of endogenous variable and parameters used in Equations [22]-[25].

Typically deficiencies in regenerative and non-regenerative wealth below threshold values causes decline in total consumption (see Equation [20]). Yet with the availability of renewable flow power, it is assumed that the decline in consumption levels caused by the deficiencies in regenerative and non-regenerative wealth can be augmented by renewable flow power. Hence, "effective" consumption ($C_{C_{(eff)}}$, $C_{E_{(eff)}}$) is introduced as per Equations [26] and [27]. The assumption is that if renewable flow power augments consumption of wealth, then the increase in population death rates, due to decline in regenerative and non-regenerative wealth below threshold values, will not be as high (see

Equation [28]). The reason is that renewable flow power can contribute towards the populations "effective" consumption needs.

$$\mathbf{C_{c(eff)}} = \boldsymbol{min}\mathbf{(1,\ (}\frac{(w_R+w_{NR})}{(w_{TH})}\mathbf{))sx_c} + \left(\frac{P_{RF}}{x_c+\kappa x_E}\right) x_c \qquad \textbf{[26]}$$

$$\mathbf{C_{E(eff)}} = \boldsymbol{min}\mathbf{(1,\ (}\frac{(w_R+w_{NR})}{(w_{TH})}\mathbf{))\ \kappa sx_E} + \left(\frac{P_{RF}}{x_c+\kappa x_E}\right) \kappa x_E \qquad \textbf{[27]}$$

The effective consumption is then used in calculating the population death rates (see Equation [28] and [29]), which effect the population growth.

$$\boldsymbol{\alpha_c = \alpha_m + \max(\ 0,\ 1-\ \frac{C_{C(eff)}}{sx_C})\ (\alpha_M - \alpha_m)} \qquad \textbf{[28]}$$

$$\boldsymbol{\alpha_E = \alpha_m + \max(\ 0,\ 1-\ \frac{C_{E(eff)}}{sx_E})\ (\alpha_M - \alpha_m)} \qquad \textbf{[29]}$$

A major drawback, of version1 is that consumption is not disaggregated between stock, and power, depending on their specific characteristics. Which leads to the assumption that population could survive on power alone. Thus, even with all regenerative and non-regenerative resources being totally exhausted, population could continue to survive on only renewable flow resources available. This drawback is addressed in version3 of the model.

2.3. Simulations and Results of HAN3DY version1.

In this initial model version1 (v1), a major assumption is that an investment to extract each of the resources is dependent on the number of commoners working on each of the sectors (only people are needed in extracting nature), but not dependent upon two other factors of production that are considered later (e.g. accumulated capital or wealth that represents machines, wealth consumption that represents power inputs). The total commoner population is then divided into three separate groups that each work to extract one of the different kinds of nature as shown in Equations [30]-[32]:

$$\boldsymbol{x_{c,R}} = \vartheta_R \boldsymbol{x_C} \qquad \textbf{[30]}$$

$$\boldsymbol{x_{c,NR}} = \vartheta_{NR} \boldsymbol{x_C} \qquad \textbf{[31]}$$

$$\boldsymbol{x_{c,RF}} = (\mathbf{1} - \vartheta_R - \vartheta_{NR})\boldsymbol{x_C}, \qquad \textbf{[32]}$$

such that $\vartheta_R + \vartheta_{NR} \leq \mathbf{1}$.

Symbol	Endogenous Variables (EV)/ Parameters (P)	Unit
$x_{c,R}$	Commoners working on regenerative nature (EV)	Person
$x_{c,NR}$	Commoners working on non-regenerative nature (EV)	Person
$x_{c,RF}$	Commoners working on renewable flow nature (EV)	Person
ϑ_R	Percentage of commoners working in Regenerative Nature extraction (P)	--
ϑ_{NR}	Percentage of commoners working in Non-Regenerative Nature extraction (P)	--

Table 2.8: Description and units of endogenous variable and parameters used in Equations [30]-[32].

A casual flow diagram of version1 of HAN3DY is shown in Figure 2.1. The black arrows indicate the direct flows, while the red arrows indicate the inputs for the extraction process. The dashed black arrows indicate information flow.

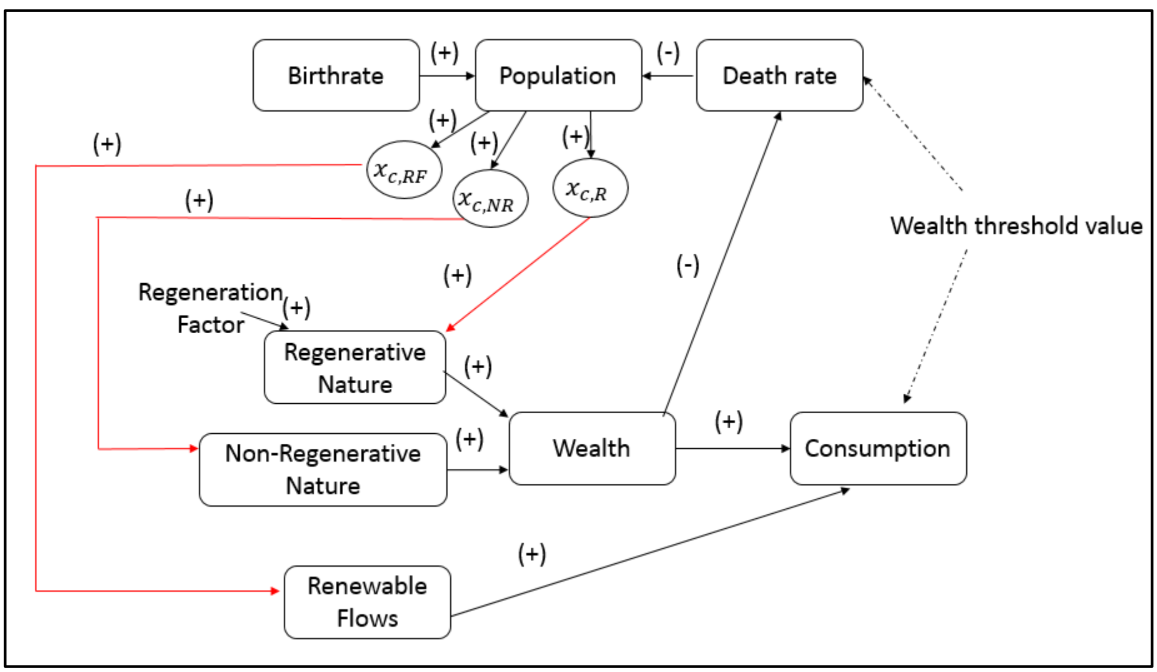


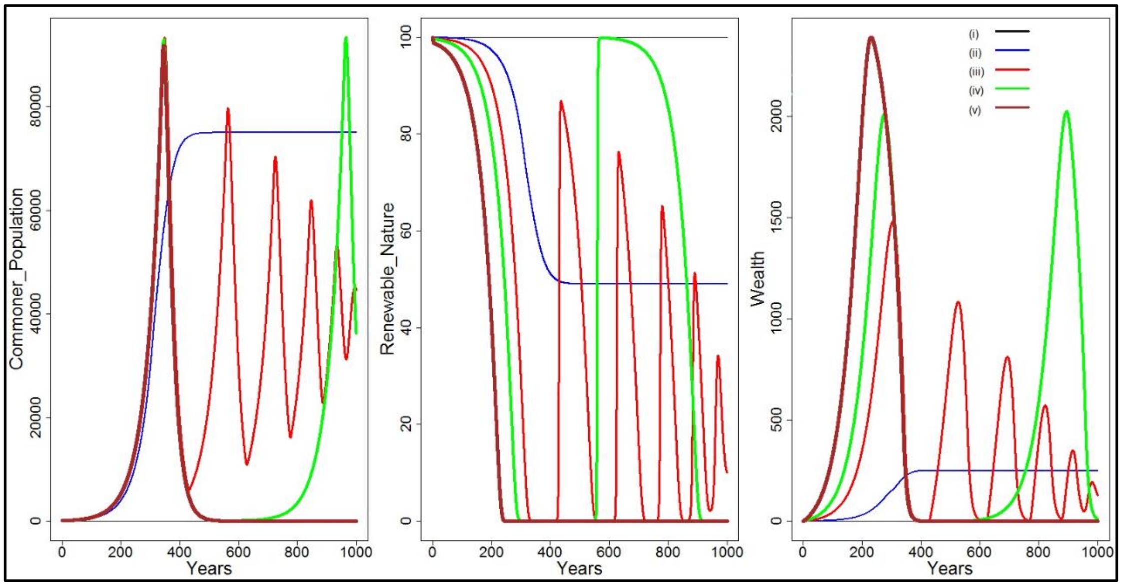
Figure 2.1: Casual flow diagram of the HAN3DY (v1) Model.

Simulations in this section have been done for an egalitarian society considering there are no elites (e.g., $x_E = 0$). Hence, there is no inequality among the population, and the total population is engaged in extracting nature.

2.3.1 Simulating the Model assuming only One Kind of Nature Available

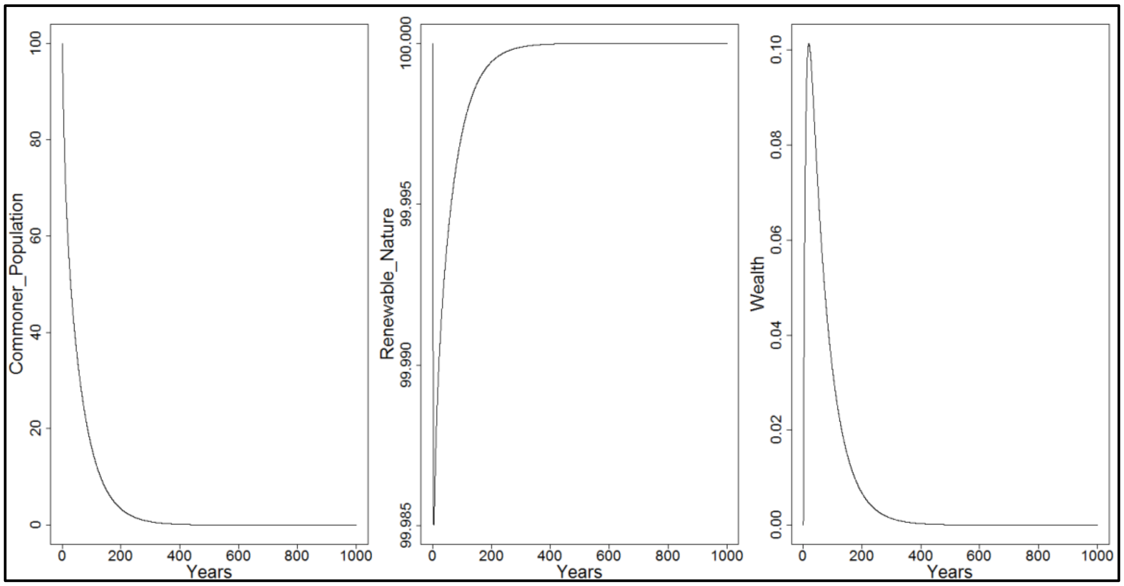
In this section, simulations are done to understand the system dynamics, when only one kind of nature is available. This allows one to understand how, each nature resource contribute towards the system dynamics independently.

Simulation 2.3.1.(i). In going through simulation results for version1 of HAN3DY model, the first simulation is to understand the parameter dynamics when there exists only regenerative resources, at different extraction rates (δ_{R}). The key parameters used are: $\mathrm{y_{NR}} = 0$, $\lambda_F = 0, \mathrm{y_R} = 100, x_{c,R} = x_C$, while other parameters are as per Appendix1, Table 1.



(a) (b) (c)

Figure 2.2: (a): Commoner Population, (b): Renewable Nature, (c) Wealth, when simulating HAN3DY v1 (equal to original HANDY model) while varying extraction rate: ($\delta^*=6.67\times10^{-6}$ person^{-1}•time^{-1}), i)$\delta=0.25\times\delta^*=1.667\times10^{-6}$, ii)$\delta=\delta^*=6.67\times10^{-6}$, iii) $\delta=3\times\delta^*=2.001\times10^{-5}$, iv) $\delta=6\times\delta^*=4.001\times10^{-5}$, v)$\delta=10\times\delta^*=6.67\times10^{-5}$.



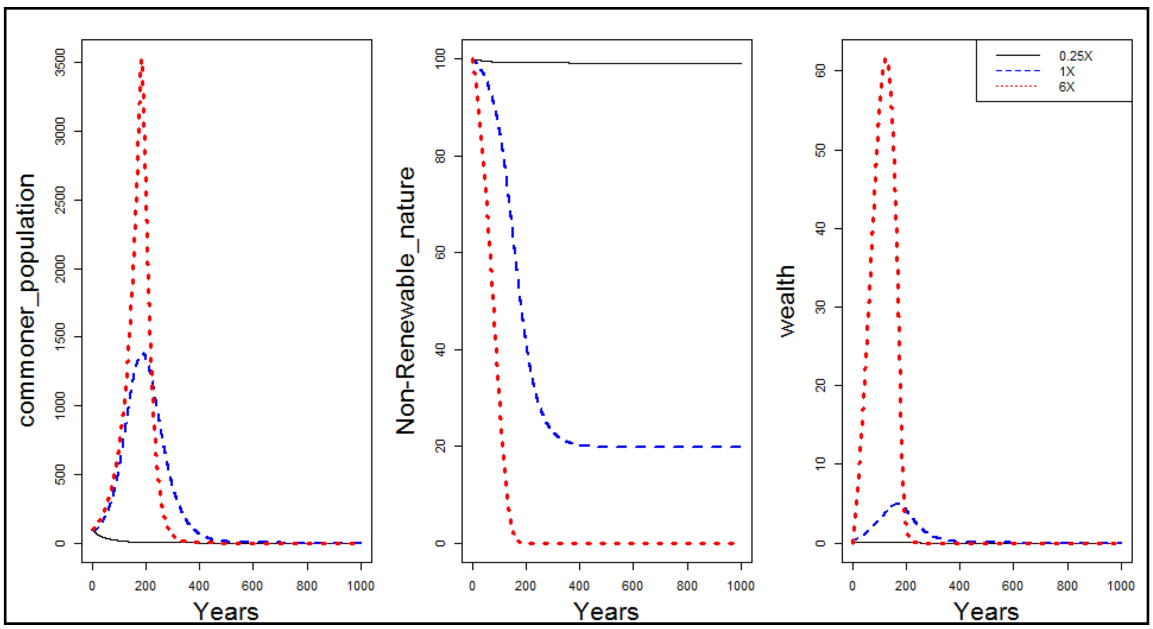
(a) (b) (c)

Figure 2.3: (a): Commoner Population, (b): Renewable Nature, (c) Wealth. A zoomed view of results in Figure 2.2 (simulation (i)), when only regenerative resources exist, and at very low extraction rate i) $\delta = 0.25\times\delta^{*}=1.667\times10^{-6}$ person^{-1}•time^{-1}.

Observations- Simulation 2.3.1.(i)

The dynamics of population, regenerative nature and wealth in respect to extraction rates are similar to the traditional HANDY dynamics. When extraction rate (δ_R) is ($\delta^{*}=6.67\text{x}10^{-6}$) (optimized extraction rate for a maximum population carrying capacity as per the HANDY model), it is seen that there is soft landing to an equilibrium value in the total population as per simulation 2.3.1(i)_ii, in Figure 2.2(a). When extraction rates are low, there is collapse in society even with plenty of resources remaining because sufficient resources are not extracted to maintain the population (as in Figure 2.3(a)). Increased levels of extraction rates results in cyclic behavior and eventual collapse (simulation 2.3.1(i)_ iii, iv,v in Figure 2.2(a)) in population. Hence, this simulation brings to light the significance of the extraction rate of nature, and its influence in the population dynamics within the system.

Simulation 2.3.1.(ii). This simulation is to understand on how the system behaves when there are only non-regenerative resources at different extraction rates (δ_{NR}). The key parameters used for the simulation are: $\lambda_{\mathrm{NR}} = 100$, $\lambda_F = 0, \mathrm{y_R} = 0, x_{c,NR} = x_C$, while other parameters are as per Appendix 1, Table 1.



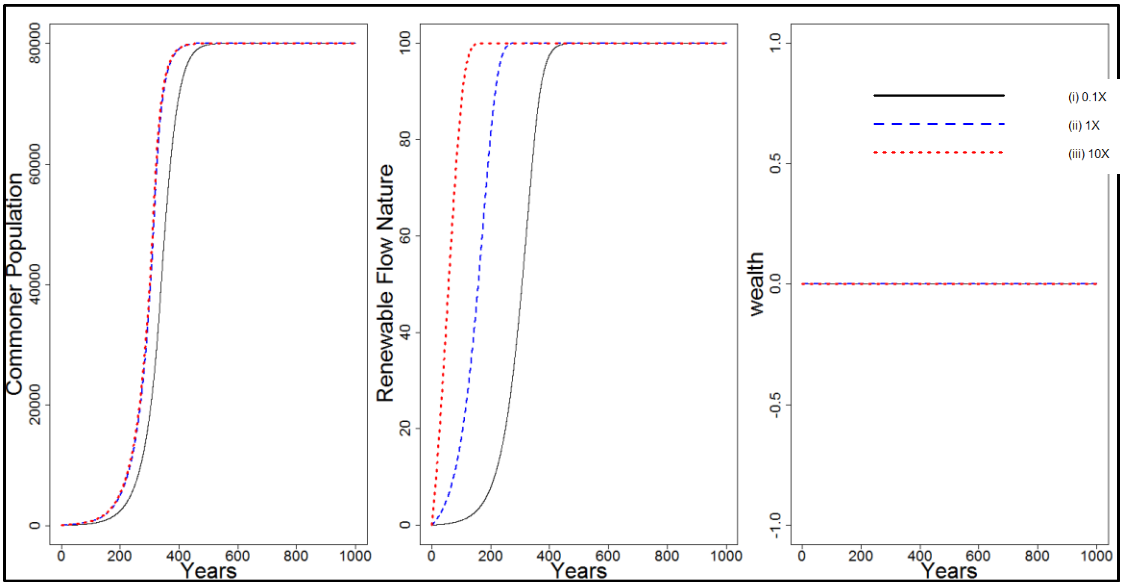
(a) (b) (c)

Figure 2.4: (a): Commoner Population, (b) Non-regenerative nature, (c) Wealth, when simulating HAN3DY v1 (when only non-regenerative resources exists) while varying the extraction rate. (δ^*= 6.67x10^{-6} person^{-1}•time^{-1}) Simulation scenarios are (i) $\delta = 0.25\times\delta^* = 1.667\text{x}10^{-6}$, ii) $\delta = 1\times\delta^* = 6.67\times10^{-6}$, iii) $\delta = 6\times\delta^* = 4.002\text{x}10^{-5}$.

Observations- Simulation 2.3.1.(ii)

When the society is only dependent on non-regenerative nature, at all extraction rates there is an eventual collapse in population. When extraction rates are low, there is collapse even without any growth in population due to the wealth generated not being sufficient to sustain the population needs (simulation 2.3.1.(ii)_i, of Figure 2.4(a)), while higher extraction rates cause initial growth in population. Yet the resource is exhausted, thus causing a later collapse in population. When extraction levels are high it causes quick accumulation of wealth, causing higher population peaks since the economic condition (accumulated wealth) is conducive for population growth. High population levels also causes quicker collapses when the accumulated wealth is exhausted. Hence, it is understood that non-regenerative resources and fast extraction of the resource results in higher initial population growth that is not sustainable by the definition of the model.

Simulation 2.3.1.(iii) This simulation is to understand the system dynamics when the system is only dependent on renewable flow resources at varying extraction rates(δ_{RF}).The key parameters used are : $\lambda_{\mathrm{NR}} = 0$, $\lambda_F = 100$, $\mathrm{y_R} = 0$, $x_{c,RF} = x_C$, while other parameters are as per Appendix 1,Table1.



(a) (b) (c)

Figure 2.5 : (a): Commoner Population, (b) Renewable Flow Nature, (c) Wealth, when simulating HAN3DY v1 (when only renewable flow resources exists) while varying the extraction rate. (δ*= 6.67x10^{-6} person^{-1}•time^{-1}) Simulation scenarios are i) $\delta = 0.1\times\delta^* = 6.67\text{x}10^{-7}$, ii)$\delta=1\times\delta^*=6.67\times10^{-6}$ iii)$\delta=10\times\delta^*=6.67\times10^{-5}$.

Observations- Simulation 2.3.1.(iii)

Although higher extraction rates cause faster growth in population (see Figure 2.5(a) , at all extraction rates the population converges to an identical population level. This identical population carrying capacity is due to the constant amount of power being generated from renewable flow resources (due to the maximum renewable flow land available is $\lambda_F = 100$, for all of the simulations) and per our version1 model, people can survive on renewable flow resource alone.

2.3.2. Combination of Regenerative and Non-Regenerative Nature.

Simulations in this section are to understand how the addition of non-regenerative resources to a system with regenerative resources would change the dynamics of the parameters in the system.

Simulation 2.3.2.(i) In this simulation, the dynamics are observed with varying extraction rates for both regenerative and non-regenerative resources, with key parameters being $\lambda_{\mathrm{NR}} = 100$, $\lambda_F = 0$, $\lambda_R = 100, x_{c,R} = 0.5x_C, x_{c,NR} = 0.5x_C$, while other parameters are as per Appendix 1, Table 1.

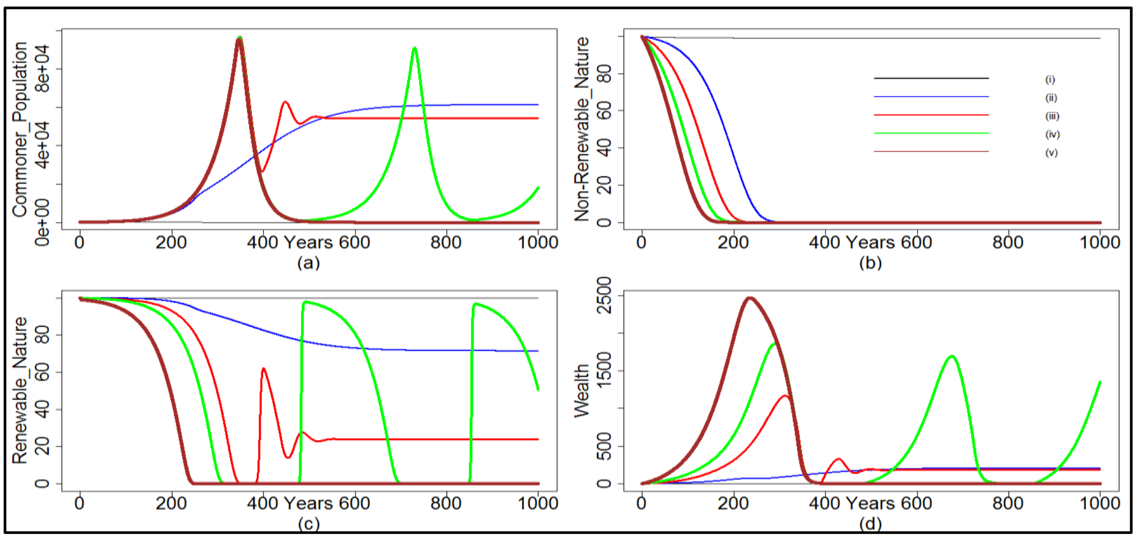


Figure 2.6: (a): Commoner population, (b): Non-regenerative nature, (c): Regenerative nature, (d): Wealth. Simulations of HAN3DY v1 assuming only regenerative and non-regenerative resources exist, while varying their extraction rates. ($\delta = \delta_R = \delta_{NR}$, $\delta * = 6.67\mathrm{x}10^{-6}$person^{-1}•time^{-1}), Simulation scenarios are, i) δ = :$0.25\times\delta^*=1.667\times10^{-6}$, ii): $\delta = \delta^* = 6.67\times10^{-6}$, iii) $\delta = 3\times\delta^*=2.001\times10^{-5}$ iv) $\delta = 6\times\delta^*=4.001\times10^{-5}$, v) $\delta = 10\times\delta^*=6.67\times10^{-5}$.

Simulation 2.3.2.(ii). This simulation is to understand the system dynamics when the total available non-regenerative resource quantities vary from a low to a high level, with key parameters: $\lambda_{NR} = 0, 100, 1000, 10000$,; $\lambda_F = 0$; $\lambda_R = 100$; $x_{c,R} = 0.5x_C$, $x_{c,NR} = 0.5x_C$, $\delta_{NR} = \delta_R = 6.6\text{x}10^{-5}$ while other parameters are as per Appendix 1, Table 1.

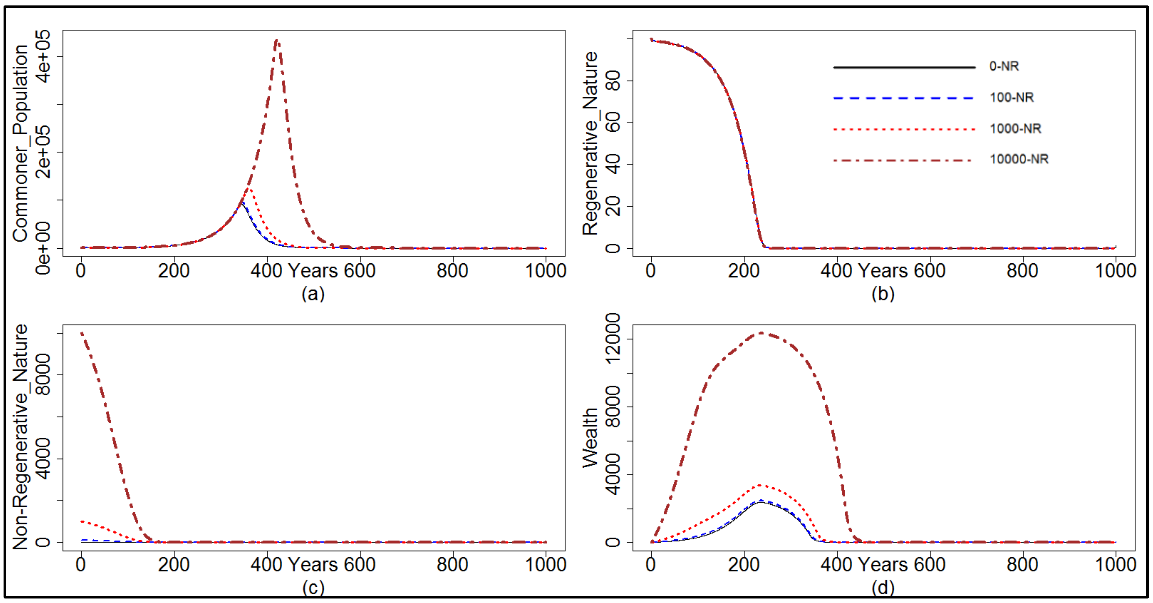


Figure 2.7: (a): Commoner population, (b): Non-regenerative nature, (c) : Regenerative nature, (d): Wealth. Simulations of HAN3DY v1 assuming only regenerative and non-regenerative resources exist, while varying the quantity of non-regenerative resources. Simulation scenarios are i) λ_{NR} =0, ii) λ_{NR} =100, iii) λ_{NR} =1000, iv) λ_{NR} =1000.

Observations- Simulation 2.3.2

Due to the declining characteristic of the non-regenerative resource and my assumption that $\delta = \delta_{R} = \delta_{NR}$, Figure 2.7 shows no equilibrium point (except at zero population) when non-regenerative resources exist. A non-zero equilibrium for population, wealth, and renewable nature can be achieved as is demonstrated via the original HANDY model and as in Figure 2.2. Equilibrium can be achieved only after all non-regenerative resources have been exhausted. Yet it is seen that the addition of this resource type to regenerative resource results in higher population peak levels and faster growth rates. For example, at very high extraction rates that cause eventual collapse, it is seen that the addition of the NR resources extends the time line until collapse Figure 2.7(a).

2.3.3. Combination of Regenerative, Non-Regenerative, and Renewable Flow Nature Available in the system.

The simulations in this section considers the availability of all three kinds of nature (regenerative, non-regenerative and renewable flows).

Simulation 2.3.3. This simulation is done to understand the dynamics of the system when all three kinds of resources exist, and when extraction rates are varied. key parameters used for these simulations are, $\lambda_{NR} = 100$, $\lambda_{F} = 100$; $y_{R} = 100$; $x_{c,R} = 0.33x_{C}$, $x_{c,NR} = 0.33x_{C}$, $x_{c,RF} = 0.33x_{C}$ while other parameters are as per Appendix 1, Table 1.

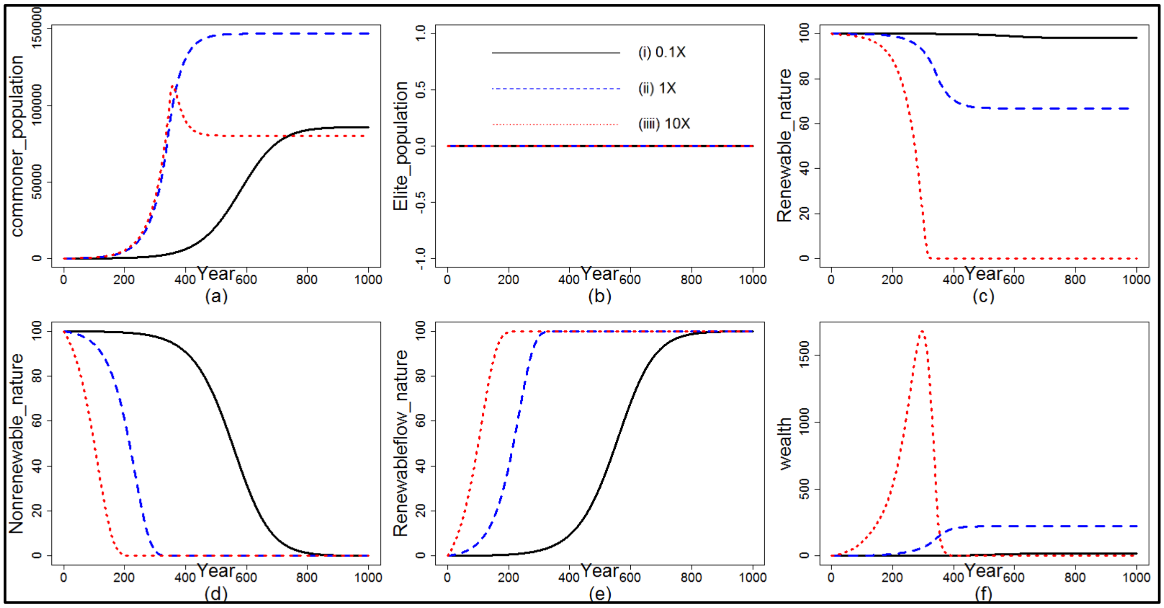


Figure 2.8: (a): Commoner Population, (b): Elite Population, (c): Renewable nature, (d): Non-regenerative Nature, (e): Renewable Flow nature, (f) Wealth, Simulations of HAN3DY v1 when all three nature resources exist, when extraction rates of regenerative, non-regenerative and renewable flows are varied. ($\delta^{*}= 6.67x10^{-6}(\text{person}^{-1} \bullet \text{time}^{-1})$. Simulation scenarios are i) $\delta = 0.1x\delta^{*} = 6.67x10^{-7}$ ii) $\delta = 1x\delta^{*} = 6.67x10^{-6}$ iii) $\delta = 1x\delta^{*} = 6.67x10^{-6}$.

Observations Simulation 2.3.3.

According to Figure 2.8(a), it is seen that version1 model cannot simulate a total collapse in population, when renewable flow resources exist. As per simulation 2.3.3.(iii) when extraction rates are high, due to the creation of wealth and power generation, population levels continue to increase, until all regenerative and non-regenerative resources are exhausted, at which point the population declines and converges to a lower population level. (which is the population level that can be accommodated by the renewable flow resources available in the system). As per simulations 2.3.3.(i) and (ii), the population levels converge to a higher equilibrium value than in simulation 2.3.3. (iii), due to the availability of regenerative resources in the system.

Chapter 3: Capital Investment

3.1 Introduction

This chapter introduces version2 of the HAN3DY model. In version1 of HAN3DY model, one assumption was that labor (the commoner population working on extracting each type of resource) is the only factor influencing production. In reality, labor alone is less efficient (with reference to work done, per unit amount of time) and in most instances insufficient for the production or extraction process. As a simple example, surface mining of a coal resource could be done in a variety of ways depending on the amount of capital being used. i) zero capital: workers digging out coal by hand, ii) minimal amount of capital: workers using spades and basic tools to dig up the coal, iii) moderate amount of capital: use of backhoes in digging up the coal, and iv) a high level of capital: the use of a bucket wheel excavator in the mining process. In general, the increased capital investment in a project can increase efficiency while reducing the labor requirement (capital substitutes for labor). Hence, due to the significance of capital in the extraction process, this chapter introduces capital to the system.

3.2 Model Development

In economics a common set of input factors that govern production (Y) are labor (L), capital (K), material usage (M) and energy (E).

$$Y = f\,(L, K, M, E)$$

In verion2 of the HAN3DY model, capital is introduced into the production function, for the extraction of nature, in addition to labor. Capital is often referred to as financial assets, which could either be working capital like cash or fixed capital, such as machinery, production equipment, factories and other manufacturing facilities that are needed to start up and conduct business even at a minimal stage. Since HAN3DY model does not consider money or cash, capital in HAN3DY is most consistently interpreted as fixed capital.

"Capital" is created by investing in it. Hence in the HAN3DY model a fraction of accumulated wealth is invested in generating capital, which is then used in the production functions of extraction. Also capital is not consumed, but it is accumulated over time. The loss of capital with age is accounted for through depreciation. Figure 3.1 illustrates the causal flow diagram of capital, within the HAN3DY model.

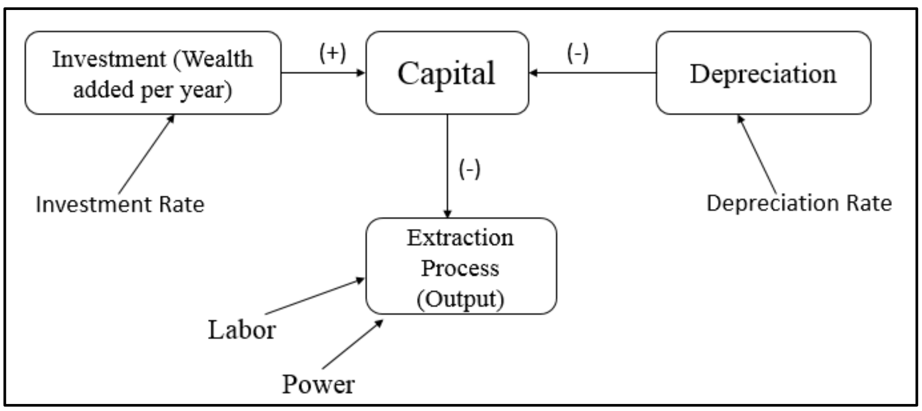


Figure 3.1: Causal flow diagram of capital in HAN3DY(v2) model.

In reference to the two kinds of nature stocks available (regenerative and non-regenerative) there are two kinds of capital present in the model. The two kinds of capital can have different generation and depreciation rates based on their characteristics.

3.2.1 Regenerative Stock Capital

Regenerative stock capital is the capital generated from regenerative nature stocks, and is governed as per Equation [33]. Regenerative stock capital is generated by investing a fraction (μ_R) of the accumulated regenerative wealth, which is converted to capital, by the wealth over capital ratio (ε) which is a parameter that dictates how much nature (stored as wealth) it takes to become one unit of fixed capital. Capital accumulation rate ($\dot{\boldsymbol{K}}_R$) is dependent on the amount of capital being generated and the depreciation of it as defined in Equation [33].

$$\dot{\boldsymbol{K}}_{\boldsymbol{R}} = \frac{\mu_{\boldsymbol{R}} \boldsymbol{w}_{\boldsymbol{R}}}{\boldsymbol{\varepsilon}} - \emptyset_{\boldsymbol{R}} \boldsymbol{K}_{\boldsymbol{R}} \qquad [33]$$

Symbol	**Endogenous Variables(EV)/ Parameters(P)**	**Unit**
K_R	Regenerative Capital (EV)	capital
$\emptyset_R$	Regenerative Capital Depreciation rate (P)	-
μ_R	Percentage of regenerative wealth invested towards capital creation (P)	-
w_R	Accumulated regenerative wealth (EV)	nature
ε	Wealth/capital ratio (P)	nature•capital^{-1}

Table 3.1: Description and units of endogenous variables and parameters used in Equation [33]

3.2.2 Non-Regenerative Stock Capital

While non-regenerative stock capital is generated from non-regenerative resources the governing equation is similar to the regenerative capital in terms of generation and depreciation.

$$\dot{K}_{NR} = \frac{\mu_{NR} w_{NR}}{\varepsilon} - \emptyset_{NR} K_{NR} \quad [34]$$

Symbol	Endogenous Variables(EV)/ Parameters(P)	Unit
K_{NR}	Non-Regenerative Capital (EV)	capital
$\emptyset_{NR}$	Non-Regenerative Capital Depreciation rate (P)	-
μ_{NR}	Percentage of non-regenerative wealth invested towards capital creation (P)	-
w_{NR}	Accumulated non-regenerative wealth (EV)	nature
ε	Wealth/capital ratio (P)	nature•capital^{-1}

Table 3.2: Description and units of endogenous variable and parameters used in equation [34].

3.2.3 New Parameters for Version2 of HAN3DY

Production Function

The production function in economics relates to the physical output of a production process to physical inputs or factors of production. The use of aggregated production function is a practice that can be traced back to the 1840's and can be developed by fitting historical economic data to relate the output in reference to the input factors (Heun, et. al, 2017). Due to the elasticity of substitution between the input factors the production function has seen various practices throughout the years with debate on the best and meaningful use of aggregated production functions. Historically the most common mainstream aggregated production function has been the Cobb-Douglass function (Heun, et. al, 2017), as per Equation [35].

$$\mathbf{Y= AK^{\alpha}L^{\beta}} \qquad \textbf{[35]}$$

Here output Y, often attempting to represent net economic output such as gross domestic product, is written as a function of traditional factors capital (K) and labor (L), total factor productivity (A), with α and β being output elasticities for capital and labor, with the constraint of $\alpha + \beta = 1$ applied to impose constant returns to scale. In Cobb-Douglass there are two assumptions i) the output elasticities being constant throughout time and ii) elasticity of substitution being fixed at one, that are criticized because the assumptions do not represent historical data trends (Heun et. al , 2017).

The Hicks elasticity of substitution between input factors x_1 and x_2, is defined as

$$\boldsymbol{\sigma_{x_1,x_2} = -\frac{\partial \ln(\frac{x_1}{x_2})}{\partial \ln(\frac{\partial y/\partial x_2}{\partial y/\partial x_1})}} \qquad \textbf{[36]}$$

The Hicks elasticity of substitution equation is used to quantify the substitution between x_1 and x_2. While $\sigma_{x_1,x_2} = 0$, indicates that x_1 and x_2 are perfect complements (e.g., x_1 cannot substitute for x_2, and vice versa), $\sigma_{x_1,x_2} = \infty$ reveals that x_1 and x_2 are perfect substitutes (e.g., an infinite amount of x_1 can substitute for an infinitesimally small amount of x_2, and vice versa). Hence, the Hicks elasticity substitution helps generalize the Cobb-Douglas equation, in introducing the Constant Elasticity of Substitution (CES) methodology addressing the above drawbacks (Heun, et. al , 2017). The generalized equation for CES is as per Equation [37], with the elasticity of substitution derived as per Equation [38].

$$\boldsymbol{Y = e^{\lambda t}[\delta_1 K^{-\rho_1} + (1 - \delta_1)L^{-\rho_1}]^{-1/\rho_1}} \qquad \textbf{[37]}$$

$$\boldsymbol{\sigma_1 = \frac{1}{(1+\rho_1)}} \qquad \textbf{[38]}$$

Symbol	**Parameters**
Y	Output
λ	Technology development factor through time
δ_1	Weighting factor for capital in production
σ_1	Elasticity of substitution
ρ_1	Model coefficient used in calculating elasticity
K	Capital
L	Labor

Table 3.3: Description of parameters used in Equations [37] and [38].

In the HAN3DY model, the total capital in the production function is considered to be the addition of the two types of capital; regenerative and non-regenerative ($K_R + K_{NR}$) since it is assumed that regenerative and non-regenerative capital are substitutable.

<u>Nature:</u>

The production function introduced in Equation [37] is applied to extraction components of all types of nature. Hence the revised equations governing nature quantities would include the CES production function. Here the extraction quantities for each nature type are dependent on both labor and capital assigned for each type of nature extraction. The revised Equations for each kind of nature are as [39], [40] and [41].

Equation governing regenerating nature (version2, HAN3DY model):

$$\dot{\boldsymbol{y}}_{\boldsymbol{R}} = \boldsymbol{\Upsilon}\mathbf{y}_{\mathbf{R}}(\boldsymbol{\lambda}_{\mathbf{R}} - \mathbf{y}_{\mathbf{R}}) - \boldsymbol{\delta}_{\mathbf{R}}\,\mathbf{y}_{\mathbf{R}}\,\boldsymbol{e}^{\boldsymbol{A_R t}}[\boldsymbol{\delta}_{\mathbf{1,R}}(\boldsymbol{K}_{\boldsymbol{R\_R}} + \boldsymbol{K}_{\boldsymbol{NR\_R}})^{-\boldsymbol{\rho}_1} + (\mathbf{1} - \boldsymbol{\delta}_{\mathbf{1,R}})\mathbf{x}_{\mathbf{c,R}}{}^{-\boldsymbol{\rho}_1}]^{-\mathbf{1}/\boldsymbol{\rho}_1} \quad \textbf{[39]}$$

Equation governing non-regenerating nature (version 2, HAN3DY model):

$$\dot{\boldsymbol{y}}_{\boldsymbol{NR}} = -\,\boldsymbol{\delta}_{\mathbf{NR}}\,\mathbf{y}_{\mathbf{NR}}\,\boldsymbol{e}^{\boldsymbol{A_{NR} t}}[\boldsymbol{\delta}_{\mathbf{1,NR}}(\boldsymbol{K}_{\boldsymbol{R\_NR}} + \boldsymbol{K}_{\boldsymbol{NR\_NR}})^{-\boldsymbol{\rho}_1} + (\mathbf{1} - \boldsymbol{\delta}_{\mathbf{1,NR}})\mathbf{x}_{\mathbf{c,NR}}{}^{-\boldsymbol{\rho}_1}]^{-\mathbf{1}/\boldsymbol{\rho}_1} \quad \textbf{[40]}$$

Equation governing renewable flow nature (version 2, HAN3DY model):

$$\dot{\boldsymbol{y}}_{\boldsymbol{RF}} = \boldsymbol{\delta}_{\mathbf{RF}}(\boldsymbol{\lambda}_{\boldsymbol{F}} - \mathbf{y}_{\mathbf{RF}})\boldsymbol{e}^{\boldsymbol{A_{RF} t}}[\boldsymbol{\delta}_{\mathbf{1,RF}}(\boldsymbol{K}_{\boldsymbol{R\_RF}} + \boldsymbol{K}_{\boldsymbol{NR\_RF}})^{-\boldsymbol{\rho}_1} + (\mathbf{1} - \boldsymbol{\delta}_{\mathbf{1,RF}})\mathbf{x}_{\mathbf{c,RF}}{}^{-\boldsymbol{\rho}_1}]^{-\mathbf{1}/\boldsymbol{\rho}_1} \quad \textbf{[41]}$$

Symbol	**Endogenous Variables (EV)/ Parameters (P)**	**Unit**
y_R	Regenerative nature (EV)	nature
y_{NR}	Non-regenerative nature (EV)	nature
y_{RF}	Renewable flow nature extracted (Land) (EV)	nature
λ_R	Regenerative nature carrying capacity (P)	nature
λ_F	Renewable flow nature carrying capacity (Maximum available land) (P)	nature

x_{cR}	Commoners working on regenerative nature (EV)	person
$x_{c,NR}$	Commoners working on non-regenerative nature (EV)	person
$x_{c,RF}$	Commoners working on renewable flow nature (EV)	person
δ_R	Extraction rate of regenerative nature (P)	$time^{-1}$
δ_{NR}	Extraction rate of non-regenerative nature (P)	$time^{-1}$
δ_{RF}	Extraction rate of renewable flow nature (Land) (P)	$time^{-1}$
Υ	Regeneration rate (P)	$nature^{-1}time^{-1}$
ρ_1	Model coefficient used for calculating elasticity of substitution between capital and labor (P)	-
t	time	time
A_R	Technology development factor for regenerating nature extraction (P)	$time^{-1}$
A_{NR}	Technology development factor for non- regenerating nature extraction (P)	$time^{-1}$
A_{RF}	Technology development factor for renewable flow nature extraction (P)	$time^{-1}$
$\delta_{1,R}$	Weighting factor for capital, in regenerative stock extraction. (P)	-
$\delta_{1,NR}$	Weighting factor for capital, in non-regenerative stock extraction. (P)	-
$\delta_{1,RF}$	Weighting factor for capital, in renewable flow nature extraction (P)	-
K_{R_R}	Regenerative capital toward, regenerative stock extraction (EV)	capital
K_{R_NR}	Regenerative capital toward, non-regenerative stock extraction (EV)	capital
K_{R_RF}	Regenerative capital toward, renewable flow nature extraction (EV)	capital
K_{NR_R}	Non-regenerative capital toward, regenerative stock extraction (EV)	capital
K_{NR_NR}	Non-regenerative capital toward, non-regenerative stock extraction (EV)	capital
K_{NR_RF}	Non-regenerative capital toward, renewable flow nature extraction (EV)	capital

Table 3.4: Description of endogenous variable and parameters and units used in Equations [39], [40] and [41].

A distinct fraction of the accumulated regenerative and non-regenerative capital is assigned to each extracting process since, capital once invested towards a particular extraction process cannot be used for another extraction process (e.g., once a coal-fired power plant is built, it is not possible to use the same materials for the power plant and convert it to a photovoltaic power plant, at least in the short term without recycling). The capital allocation for each extraction processes are:

$$\boldsymbol{K_{R\_R} = f_{R\to R} K_R} \quad \textbf{[42]}$$

$$\boldsymbol{K_{R\_NR} = f_{R\to NR} K_R} \quad \textbf{[43]}$$

$$\boldsymbol{K_{R\_RF} = (1 - f_{R\to R} - f_{R\to NR}) K_R} \quad \textbf{[44]}$$

$$\boldsymbol{K_{NR\_R} = f_{NR\to R} K_{NR}} \quad \textbf{[45]}$$

$$\boldsymbol{K_{NR\_NR} = f_{NR\to NR} K_{NR}} \quad \textbf{[46]}$$

$$\boldsymbol{K_{NR\_RF} = (1 - f_{NR\to R} - f_{NR\to NR}) K_{NR}} \quad \textbf{[47]}$$

Such that : $f_{R\to R} + f_{R\to NR} \le 1$, $f_{NR\to R} + f_{NR\to NR} \le 1$

Symbol	Parameters (P)	Unit
$f_{R \to R}$	Fraction of regenerative capital towards regeneration stock extraction	-
$f_{R \to NR}$	Fraction of regenerative capital towards non-regeneration stock extraction	-
$f_{R \to RF} = 1 - f_{R \to R} - f_{R \to NR}$	Fraction of regenerative capital towards renewable flow nature extraction	-
$f_{NR \to R}$	Fraction of non-regenerative capital towards regeneration stock extraction	-
$f_{NR \to NR}$	Fraction of non-regenerative capital towards non-regeneration stock extraction	-
$f_{NR \to RF} = 1 - f_{NR \to R} - f_{NR \to NR}$	Fraction of non-regenerative capital towards renewable flow nature extraction	-

Table 3.5: Description of and parameters and units used in Equations [42]–[47].

Wealth

The accumulated regenerative and non-regenerative wealth Equations [48]–[49] are revised to include the revised nature extraction components of Equations [39]-[41], and the fraction of wealth invested towards generating capital. As illustrated in Figure 3.2 the accumulation rate of wealth is increased by the extractions and it is decreased by the consumption by the population and the investment towards capital.

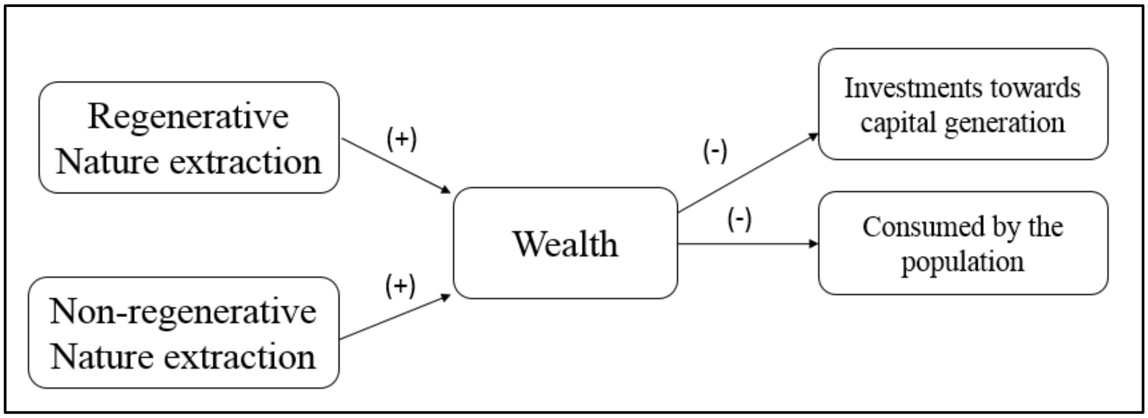


Figure 3.2: Casual flow diagram of wealth in version2 of HAN3DY

$$\dot{\boldsymbol{w}}_{\boldsymbol{R}} = \boldsymbol{\delta}_{\mathbf{R}}\, \mathbf{y}_{\mathbf{R}}\, \boldsymbol{e}^{\boldsymbol{A}_{\boldsymbol{R}}\boldsymbol{t}}[\boldsymbol{\delta}_{\mathbf{1},\mathbf{R}}(\boldsymbol{K}_{\boldsymbol{R\_R}} + \boldsymbol{K}_{\boldsymbol{NR\_R}})^{-\boldsymbol{\rho}_{\mathbf{1}}} + (\mathbf{1} - \boldsymbol{\delta}_{\mathbf{1},\mathbf{R}})\mathbf{x}_{\mathbf{c},\mathbf{R}}{}^{-\boldsymbol{\rho}_{\mathbf{1}}}]^{-\mathbf{1}/\boldsymbol{\rho}_{\mathbf{1}}} - \boldsymbol{C}_{\boldsymbol{C},\boldsymbol{R}} - \boldsymbol{C}_{\boldsymbol{E},\boldsymbol{R}} - \mu_R \boldsymbol{w}_{\boldsymbol{R}} \quad \textbf{[48]}$$

$$\dot{\boldsymbol{w}}_{\boldsymbol{NR}} = \boldsymbol{\delta}_{\mathbf{NR}}\, \mathbf{y}_{\mathbf{NR}}\, \boldsymbol{e}^{\boldsymbol{A}_{\boldsymbol{NR}}\boldsymbol{t}}[\boldsymbol{\delta}_{\mathbf{1},\mathbf{NR}}(\boldsymbol{K}_{\boldsymbol{R\_NR}} + \boldsymbol{K}_{\boldsymbol{NR\_NR}})^{-\boldsymbol{\rho}_{\mathbf{1}}} + (\mathbf{1} - \boldsymbol{\delta}_{\mathbf{1},\mathbf{NR}})\mathbf{x}_{\mathbf{c},\mathbf{NR}}{}^{-\boldsymbol{\rho}_{\mathbf{1}}}]^{-\mathbf{1}/\boldsymbol{\rho}_{\mathbf{1}}} - \mathbf{C}_{\boldsymbol{C},\boldsymbol{NR}} - \boldsymbol{C}_{\boldsymbol{E},\boldsymbol{NR}} - \mu_{NR} \boldsymbol{w}_{\boldsymbol{NR}} \quad \textbf{[49]}$$

Symbol	**Endogenous Variables (EV)/ Parameters (P)**	**Unit**
w_R	Accumulated regenerative wealth (EV)	nature
w_{NR}	Accumulated non-regenerative wealth (EV)	nature
$C_{C,R/NR}$	Consumption of commoners, regenerative/ non-regenerative (EV)	nature•time^{-1}
$C_{E,R/NR}$	Consumption of elites, regenerative/ non-regenerative (EV)	nature
μ_R	Percentage of regenerative wealth invested towards capital generation (P)	-
μ_{NR}	Percentage of non-regenerative wealth invested towards capital generation (P)	-

Table 3.6: Description of additional endogenous variables and parameters and units used in Equations [48] and [49].

Population, Power Generation, Consumption

The equations for population, power generated from renewable flow resources, as well as the consumption by the population remains the same as in version1. The death rates in version1, of HAN3DY are a function of wealth accumulation, and the amount of renewable flow power in the system. In version2, extracted nature is used to generate both wealth and capital. Since capital available is also an indicator of the economic wellbeing in a society, the death rates are modified in version2, to include capital per capita as per equations [50] and [51]. For example, food could be considered as a physical stock (wealth) which helps grow population by reducing the death rate, but also hospitals (e.g., capital) are important components in society that can help reduce death rates. Thus, in version2, death rates can increase if there is not enough wealth per person as well as if there is not enough capital per person. The most limiting factor, wealth or capital, is used to dictate the death rate.

$$\alpha_c = \alpha_m + \max\left(0, 1 - \frac{C_{C(eff)}}{sx_C}, 1 - \frac{K_R+K_{NR}}{K_{TH}}\right)(\alpha_M - \alpha_m) \quad [50]$$

$$\alpha_E = \alpha_m + \max\left(0, 1 - \frac{C_{E(eff)}}{sx_E}, 1 - \frac{(K_R+K_{NR})\kappa}{K_{TH}}\right)(\alpha_M - \alpha_m) \quad [51]$$

$$K_{TH} = \rho_K x_C + \kappa \rho_K x_E \quad [52]$$

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
α_C, α_E	Death rates of commoners and elites, respectively (EV)	time^{-1}
K_{TH}	Capital threshold value (P)	Capital
ρ_K	Threshold capital per capita (P)	capital•person^{-1}
ρ_{KR}	Threshold regenerative capital per capita (P)	capital•person^{-1}
ρ_{KNR}	Threshold non-regenerative capital per capita (P)	capital•person^{-1}

Table 3.7: Description of additional endogenous variables, parameters and units used in Equations [50] to [52].

The casual flow diagram of version2 of HAN3DY is shown in Figure 3.1. The black arrows indicate the direct flows, while the red arrows indicate the inputs for the extraction process. The dashed black arrows indicate information flow.

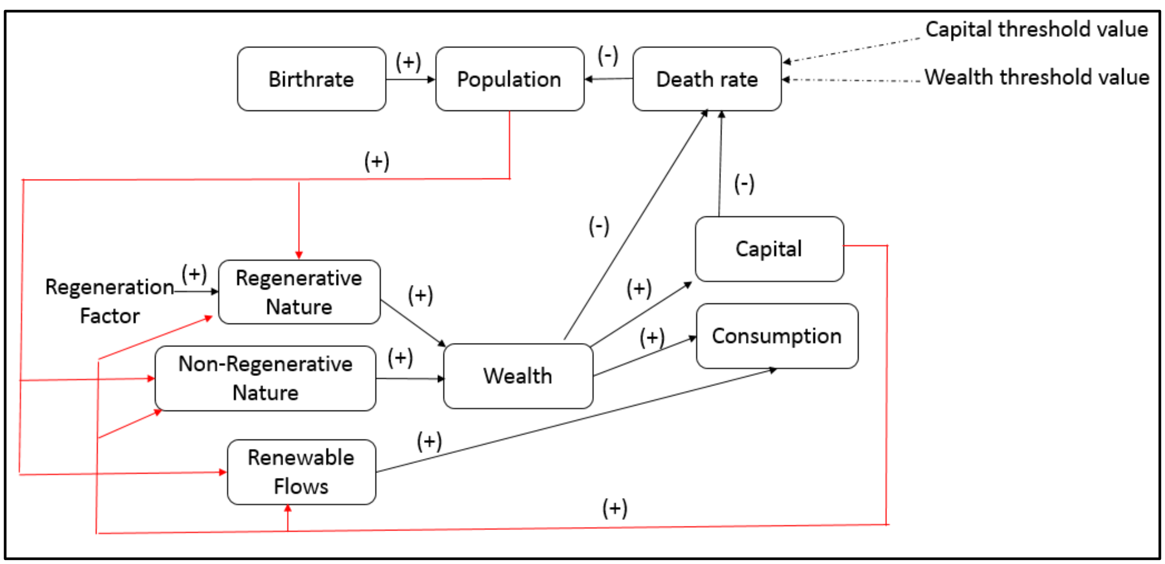


Figure 3.3: The casual flow diagram of version2 of HAN3DY

3.3. Simulation Results of HAN3DY version2

The simulation has been done for an egalitarian society considering there are no elites. Also in version1 of the model, initial accumulated wealth was zero. Yet in version2 since capital is an input parameter of extraction, the initial values of either wealth or capital needs to be greater than zero. In this version it is assumed that there exists initial wealth. Also the weighting factors between capital and labor terms in the CES production function are assumed to be equal ($\boldsymbol{\delta_{1,\mathrm{R}} = \delta_{1,\mathrm{NR}} = \delta_{1,\mathrm{RF}} = 0.5}$). The elasticity of substitution factors in the production function are as per published data on Germany ($\sigma_1 = 0.7931$) (Kemfert, 2000).

Simulation 3.3.1: This simulation is to understand the system dynamics of the parameters with varying fractions of accumulated wealth invested in generating capital. Here the nature extraction rates are kept constant, ($\boldsymbol{\delta_{\mathrm{R}}} = 6x10^{-5}\mathrm{person}^{-1} \bullet \mathrm{time}^{-1}$) , $\boldsymbol{\delta_{\mathrm{NR}}} = \boldsymbol{\delta_{\mathrm{RF}}} = 6x10^{-5}\mathrm{person}^{-1} \bullet \mathrm{time}^{-1}$) , while key parameters being, $\lambda_{\mathrm{NR}} = 100$, $\lambda_F = 100$, $\lambda_{\mathrm{R}} = 100$, $x_{c,R} = 0.33x_C$, $x_{c,NR} = 0.33x_C$, $x_{c,RF} = 0.33x_C$, $w_{R(initial)} = 10$, $w_{NR(initial)} = 10$, (other parameters as per Appendix 1,Table 2).

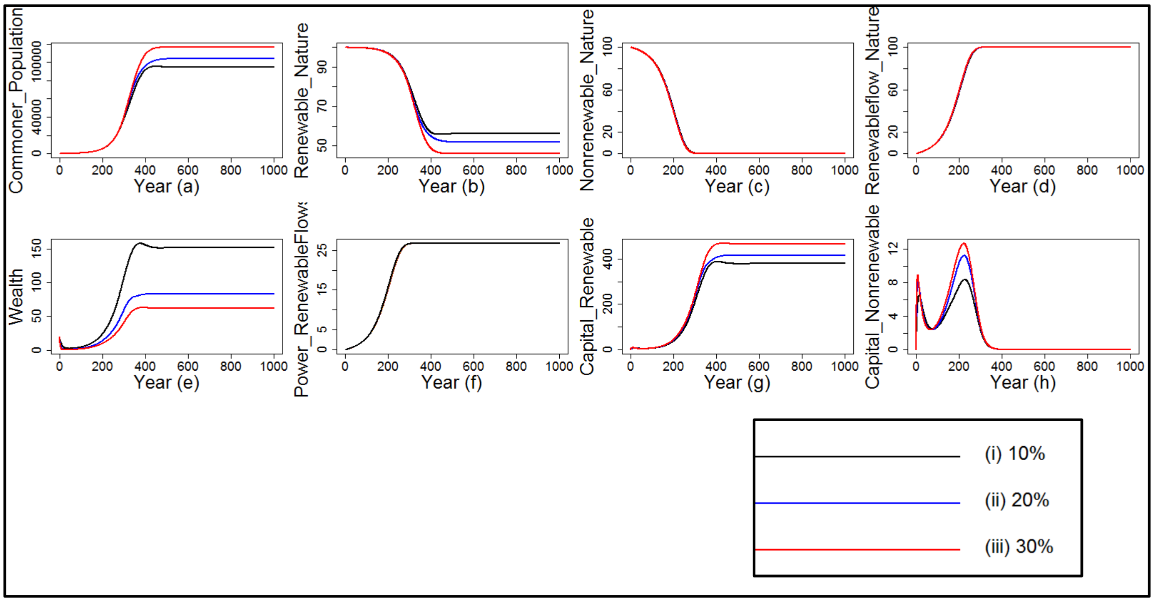


Figure 3.4: (a): Commoner Population, (b): Renewable nature, (c): Non-regenerative Nature, (d): Renewable Flow nature, (e): Wealth (f): Power Renewable Flows, (g) Capital Renewable resource, (h): Capital Nonrenewable resource. Simulations of HAN3DY v2, with varying levels of investment towards capital generation. ($\mu_R = \mu_{NR}$, Percentage of accumulated wealth invested towards capital generation) Simulation scenarios i)$\mu_R = \mu_{NR} = 10\%$, ii) $\mu_R = \mu_{NR} = 20\%$, iii)$\mu_R = \mu_{NR} = 30$

Observations- Simulation 3.3.1

As per the simulation results , when accumulated wealth invested towards capital generation is increased, the population carrying capacity increases (see Figure 3.4(a)), although the accumulated wealth in the system decreases (see Figure3.4 (e)). Capital plays two roles in the system, i) it is an input parameter to the extraction function, and ii) accumulated capital is an economic indicator, which reflects on the death rate. The system parameters are such that the available renewable flow power keeps the effective consumption rate at the subsistence levels, hence, the only limiting factor towards population growth for the above simulations are capital availability in the system.

Chapter 4: Power Investment

4.1 Introduction

This chapter introduces version3 of HAN3DY model, the final version of the model in this thesis. In version3, the scope is further expanded by the addition of energy as a factor of the production function. Also, the total consumption by the population is disaggregated into the consumption of physical stocks and energy, with relevant modifications to HAN3DY version2 model equations and parameters.

4.2 Model Development

Energy is an integral part of the modern society, and has played a significant role in the development of civilization ever since fire was discovered. Through the course of time humans have learned to exploit more and more sources of energy, and our modern economy is extremely dependent on energy consumption. Energy is generated through a variety of technologies and resources (regenerative, non-regenerative and renewable flows) and takes different forms of end-use energy carriers (e.g., gasoline, electricity) and energy services (heat, mechanical power, light, transportation).

While many mainstream economic models do not use energy as a factor in growth, resource economists often use energy as a part of resources in their growth models (Stern, 2010). The role of energy in economic production is in line with basic physical principles. As per the mass balance principle, in order to obtain a given amount of output, greater or equal amount of inputs are required with the residual being pollution or waste. The second law of thermodynamics, implies the use of energy in transformation of matter (Stern, 2010). Since all production processes involve transformation or movement of matter resulting in an output, energy is an important factor in the production of any good or service. Hence a production function in a model can take into account the input factors of labor (L), capital (K) and power (P): $Y = f\,(L, K, P)$. In this thesis, I specifically refer to power defined as energy consumed per unit time.

The HAN3DY model assumes all three nature resources can generate power, or energy per unit time to be consumed during each time step. While renewable flows can only generate power, regenerative resources (e.g., biomass, etc.) and the non-regenerative resources (e.g., coal, petroleum, natural gas) can be converted to energy stocks of different forms, such as solid, liquid, or gaseous fuels that can be consumed at some rate to produce power, such as via a power plant or other technology. In the HAN3DY model, all forms of power are aggregated together. It is also assumed that the power generated cannot be stored or accumulated and needs to be consumed. Since the consumption and allocation of natural resources in the differential equations affects the rates of consumption and accumulation of non-regenerative and regenerative nature, it represents power, not energy, since the calculations are effectively energy generated and used per unit time.

In the HAN3DY model, power can be generated and consumed from regenerative and non-regenerative natural resources that have been accumulated as wealth. In the model it is assumed that a fraction of wealth is allocated towards power generation.

The casual flow diagram of wealth in version3 of HAN3DY model is as Figure 4.1:

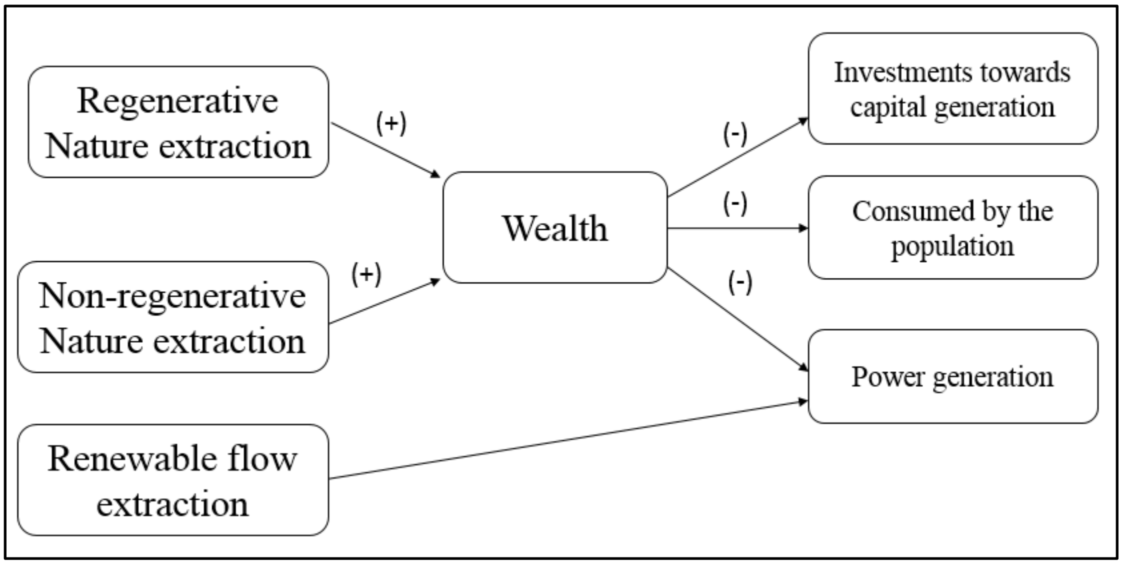


Figure 4.1: Casual flow diagram of wealth in version3 of HAN3DY

4.2.1 New Parameters

Production function

With the addition of power into the production function, the generalized production function for three factors is generated by nesting the two initial factors of production (capital and labor) in Equation [24] with power. The nested production function is as Equation [61] (Heun et. al , 2017).

$$\boldsymbol{Y = e^{\lambda t}\{\delta_2[\delta_1 K^{-\rho_1} + (1-\delta_1)L^{-\rho_1}]^{\rho_2/\rho_1} + (1-\delta_2)P^{-\rho_2}\}^{-1/\rho_2}} \quad \textbf{[53]}$$

$$\boldsymbol{\sigma_1 = \frac{1}{(1+\rho_1)}} \quad \textbf{[54]}$$

$$\boldsymbol{\sigma_2 = \frac{1}{(1+\rho_2)}} \quad \textbf{[55]}$$

Symbol	Parameters
Y	Output
λ	Technology development factor
δ_1	Weighting factor for production between capital and labor
δ_2	Weighting factor for production between (capital and labor) and Power
σ_1	Elasticity of substitution between capital and labor
σ_2	Elasticity of substitution between (capital and labor) and power
ρ_1	Coefficients used in calculating elasticity between capital and labor
ρ_2	Coefficients used in calculating elasticity between (capital and labor) and power
K	Capital
L	Labor
P	Power

Table 4.1: Description of parameters used in Equations [53]-[55]

Nature

To include power as a factor input in the production functions, the extraction component of the nature equations were revised as per [56], [57] and [58]. Here the extraction component is a function of labor, capital and power. The weighting factors between the input parameters and should satisfy the inequality 0≤ $\delta_{1,\mathrm{R}}, \delta_{2,\mathrm{R}}, \delta_{1,\mathrm{NR}}, \delta_{2,\mathrm{R}}, \delta_{1,\mathrm{RF}}, \delta_{2,\mathrm{RF}}, \leq 1$.

Regenerative Nature

$$\dot{y}_R = \Upsilon y_R(\lambda_R - y_R) - \delta_R\, y_R\, e^{A_R t}\{\delta_{2,R}[\delta_{1,R}(K_{R_R} + K_{NR_R})^{-\rho_1} + (1 - \delta_{1,R})x_{c,R}{}^{-\rho_1}]^{\frac{\rho_2}{\rho_1}} + (1 - \delta_{2,R})P_{I\_R}{}^{-\rho_2}\}^{\frac{-1}{\rho_2}} \quad [56]$$

Non-regenerative Nature

$$\dot{y}_{NR} = -\delta_{NR}\, y_{NR}\, e^{A_{NR} t}\{\delta_{2,NR}[\delta_{1,NR}(K_{R_{NR}} + K_{NR_{NR}})^{-\rho_1} + (1 - \delta_{1,NR})x_{c,NR}{}^{-\rho_1}]^{\frac{\rho_2}{\rho_1}} + (1 - \delta_{2,NR})P_{I\_NR}{}^{-\rho_2}\}^{-1/\rho_2} \quad [57]$$

Renewable flow Nature

$$\dot{y}_{RF} = \delta_{RF}(\lambda_F - y_{RF})e^{A_{RF} t}\{\delta_{2,RF}[\delta_{1,RF}(K_{R_{RF}} + K_{NR_{RF}})^{-\rho_1} + (1 - \delta_{1,RF})x_{c,RF}{}^{-\rho_1}]^{\frac{\rho_2}{\rho_1}} + (1 - \delta_{2,NR})P_{I\_RF}{}^{-\rho_2}\}^{-1/\rho_2} \quad [58]$$

Similar in structure to version2 of HAN3DY, version3 allows the production function to substitute between labor, capital and power inputs based on the elasticity parameters.

Symbol	Endogenous Variables (EV)/ Parameters (P)	Unit
y_R	Regenerative nature (EV)	nature
y_{NR}	Non-regenerative nature (EV)	nature
y_{RF}	Renewable flow nature extracted (Land) (EV)	nature
λ_R	Regenerative nature carrying capacity (P)	nature
λ_F	Renewable flow nature carrying capacity (Maximum available land) (P)	nature
x_{cR}	Commoners working on regenerative nature (EV)	person
$x_{c,NR}$	Commoners working on non-regenerative nature (EV)	person
$x_{c,RF}$	Commoners working on renewable flow nature (EV)	person
δ_R	Extraction rate of regenerative nature (P)	$time^{-1}$
δ_{NR}	Extraction rate of non-regenerative nature (P)	$time^{-1}$
δ_{RF}	Extraction rate of renewable flow nature (Land). (P)	$time^{-1}$
Υ	Regeneration rate (P)	$nature^{-1}time^{-1}$
ρ_1/ρ_2	Parameter used for calculating elasticity of substitution (P0	-
t	time	time
A_R	Technology development factor for regenerating nature extraction (P)	$time^{-1}$
A_{NR}	Technology development factor for non- regenerating nature extraction (P)	$time^{-1}$

A_{RF}	Technology development factor for renewable flow nature extraction (P)	$time^{-1}$
$\delta_{1,\mathrm{R}}$	Weighting factor for capital, in regenerative stock extraction. (P)	-
$\delta_{1,\mathrm{NR}}$	Weighting factor for capital, in non-regenerative stock extraction. (P)	-
$\delta_{1,\mathrm{RF}}$	Weighting factor for capital, in renewable flow nature extraction (P)	-
K_{R_R}	Regenerative capital toward, regenerative stock extraction (EV)	capital
K_{R_NR}	Regenerative capital toward, non-regenerative stock extraction (EV)	capital
K_{R_RF}	Regenerative capital toward, renewable flow nature extraction (EV)	capital
K_{NR_R}	Non-regenerative capital toward, regenerative stock extraction (EV)	capital
K_{NR_NR}	Non-regenerative capital toward, non-regenerative stock extraction (EV)	capital
K_{NR_RF}	Non-regenerative capital toward, renewable flow nature extraction (EV)	capital
P_{I_R}	Power input in extracting regenerative resources	nature
P_{I_NR}	Power input in extracting non-regenerative resources	nature
P_{I_RF}	Power input in extracting renewable flow resources	nature

Table 4.2: Description of endogenous variables, parameters and units used in Equations [56], [57] and [58]

Wealth

Equations governing regenerative and non-regenerative wealth are as in [59] and [60]. The accumulation rate of regenerative and non-regenerative stock wealth ($\dot{w}_R, \dot{w}_{NR}$) is dependent on the revised extraction quantities using the three factor CES production function as well as the allocation of wealth to three purposes: consumption by both elites and commoners, quantities invested towards capital creation, and quantities used for power generation (Figure 4.1).

$$\dot{w}_R = \delta_R\, y_R\, e^{A_R t}\{\delta_{2,R}[\delta_{1,R}(K_{R_R} + K_{NR_R})^{-\rho_1} + (1-\delta_{1,R})x_{c,R}{}^{-\rho_1}]^{\frac{\rho_2}{\rho_1}} + (1-\delta_{2,R})P_{I\_R}{}^{-\rho_2}\}^{-\frac{1}{\rho_2}} - C_{C,R} - C_{E,R} - \mu_R w_R - \eta_R w_R \qquad [59]$$

$$\dot{w}_{NR} = \delta_{NR}\, y_{NR}\, e^{A_{NR} t}\{\delta_{2,NR}[\delta_{1,NR}(K_{R_{NR}} + K_{NR_{NR}})^{-\rho_1} + (1-\delta_{1,NR})x_{c,NR}{}^{-\rho_1}]^{\frac{\rho_2}{\rho_1}} + (1-\delta_{2,NR})P_{I\_NR}{}^{-\rho_2}\}^{-\frac{1}{\rho_2}} - C_{C,NR} - C_{E,NR} - \mu_{NR} w_{NR} - \eta_{NR} w_{NR} \qquad [60]$$

Such that : $\mu_{NR} + \eta_{NR} < 1$, $\mu_R + \eta_R < 1$

Symbol	Endogenous Variables (EV)/ Parameters (P)	Unit
w_R	Accumulated regenerative wealth (EV)	nature
w_{NR}	Accumulated non-regenerative wealth (EV)	nature
$C_{C,R/NR}$	Consumption of commoners, regenerative/ non-regenerative (EV)	nature
$C_{E,R/NR}$	Consumption of elites, regenerative/ non-regenerative (EV)	nature
μ_R	Fraction of regenerative wealth invested towards capital generation (P)	-
μ_{NR}	Fraction of non-regenerative wealth invested towards capital generation (P)	-
η_R	Fraction of regenerative wealth converted towards power generation ($0 \le \eta_R \le 1$) (P)	-
η_{NR}	Fraction of non-regenerative wealth converted towards power generation ($0 \le \eta_{NR} \le 1$) (P)	-

Table 4.3 Description of endogenous variables, parameters and units used in Equation [59] and [60]

Power Generation

The power generated from renewable flow resource is as equation [19] of HAN3DY version1.

$$\boldsymbol{P_{RF} = I y_{RF}} \quad \textbf{[19]}$$

Since the power generated in HAN3DY through various resources is considered to be of the same form, the total power generated is the addition of power generated from the renewable flow resource ($\boldsymbol{P_{RF}}$), regenerative resource ($\boldsymbol{\eta_R w_R}$) and the non-regenerative resource ($\boldsymbol{\eta_{NR} w_{NR}}$).

$$\boldsymbol{P = P_{RF} + \eta_R w_R + \eta_{NR} w_{NR}} \quad \textbf{[61]}$$

Consumption

In version1 and version2 of HAN3DY, the total effective consumption needs of the population were in the form of physical stock, with the power generated from renewable flow resource contributing towards augmenting deficiencies below threshold values. A drawback of versions1 and 2 was the above assumption, which allowed the population to sustain on renewable flows alone. In version3 of HAN3DY consumption is disaggregated as physical stock consumption and power consumption. Physical stock consumption equations are same as version1, and denoted in Equations [22]–[25]. While the total consumption is governed by Equations [20] and [21]. Here the idea of effective consumption is not used.

Equations [62] and [63] are introduced to address the power consumption needs of the commoners and elites. Here a power threshold ($\boldsymbol{P_{TH}}$) value is introduced, below which power consumption reduces linearly as seen in Figure 4.2. The subsistence power per capita is $\boldsymbol{s_P}$. Hence when power available in the system is greater than the threshold values, $\mathbf{s}_{\boldsymbol{P}}\mathbf{x}_{\boldsymbol{C}}$ and $\boldsymbol{\kappa}\mathbf{s}_{\boldsymbol{P}}\mathbf{x}_{\boldsymbol{C}}$ are the power consumption for commoners and elites, respectively. In Equation [64] ρ_P is the threshold power consumption per capita.

$$\mathbf{C_{C,P}} = \boldsymbol{min}(\mathbf{1}, (\frac{\boldsymbol{P}}{\boldsymbol{P_{\_th}}}) \ )\mathbf{s}_{\boldsymbol{P}}\mathbf{x}_{\boldsymbol{C}} \qquad \textbf{[62]}$$

$$\mathbf{C_{E,P}} = \boldsymbol{min}(\mathbf{1}, (\frac{\boldsymbol{P}}{\boldsymbol{P_{\_th}}}) \ ) \ \boldsymbol{\kappa}\mathbf{s}_{\boldsymbol{P}}\mathbf{x}_{\boldsymbol{C}} \qquad \textbf{[63]}$$

$$\boldsymbol{P_{\_th}} = \boldsymbol{\rho_P}\mathbf{x}_{\boldsymbol{C}} + \boldsymbol{\kappa\rho_P}\mathbf{x}_{\boldsymbol{P}} \qquad \textbf{[64]}$$

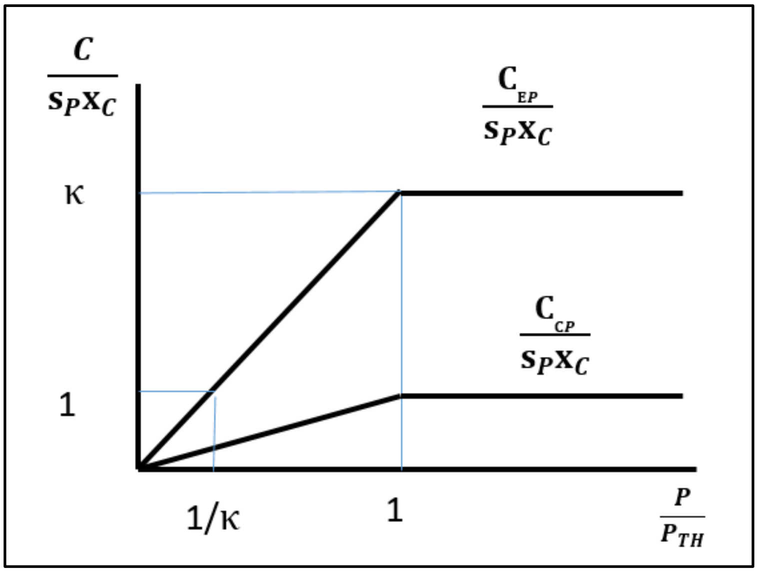


Figure 4.2: Per capita power consumption rate for elites and commoners as a function of total power in the system. Consumption rate reduces when $\frac{P}{P_{\_th}}$ <1.

Symbol	Endogenous Variables(EV)/ Parameters(P)	Units
$C_{C,P}$	Total power consumption by commoners	nature•time^{-1}
$C_{E,P}$	Total power consumption by elites	nature•time^{-1}
$P_{\_th}$	Threshold power	nature•time^{-1}
P	Power generated	nature•time^{-1}
ρ_P	Power threshold usage levels	nature•person^{-1}•time^{-1}
s_P	Power usage per capita	nature•person^{-1}•time^{-1}

Table 4.4: Description of endogenous variables, parameters and units used in Equations [62]-[64].

It is assumed that the elites consume **κ** times more power than the commoners. Power is not only used for consumption, it is also needed as an input in the extraction processes. The allocation of power between consumption and investment needs to be defined. In the HAN3DY model, the consumption needs of the population has been prioritized, and only the excess power generated over the consumption requirements goes into the extraction processes. Hence, power towards extraction processes of nature are governed by Equations [65]-[68].

$$\boldsymbol{P_I = if\left(P \geq \left(C_{C,P} + C_{E,P}\right), P - \left(C_{C,P} + C_{E,P}\right)\right), else\ P_I = 0} \qquad \textbf{[65]}$$

$$\mathbf{P}_{\boldsymbol{I\_R}} = \upsilon_{\boldsymbol{R}}\boldsymbol{P_I} \qquad \textbf{[66]}$$

$$\mathbf{P}_{\boldsymbol{I\_NR}} = \upsilon_{\boldsymbol{NR}}\boldsymbol{P_I} \qquad \textbf{[67]}$$

$$\mathbf{P}_{\boldsymbol{I\_RF}} = (\mathbf{1} - \upsilon_{\boldsymbol{R}} - \upsilon_{\boldsymbol{NR}})\boldsymbol{P_I} \qquad \textbf{[68]}$$

The power input towards each extraction process is a fraction of the excess power generated beyond the consumption levels of the population, and can be determined in the model.

Symbol	Endogenous Variables (EV)/ Parameters (P)	Units
P_I	Total power "invested" as an input to extraction processes (EV)	nature•time^{-1}
P_{I_R}	Power towards regenerative resource extraction processes (EV)	nature•time^{-1}
P_{I_NR}	Power towards non-regenerative resource extraction processes (EV)	nature•time^{-1}
P_{I_RF}	Power towards renewable flow land extraction, or development processes (EV)	nature•time^{-1}
υ_R	Fraction of total power towards extraction going into regenerative extractions (P)	-
υ_{NR}	Fraction of total power towards extraction going into non-regenerative extractions (P)	-
Constraint Equation	$P_I = P_{I\_R} + P_{I\_NR} + P_{I\_RF}$	

Table 4.5: Description of endogenous variables, parameters and units used in equation [65]-[68].

Population

Population characteristic is similar to the HANDY model

$$\dot{x}_C = \beta_C x_C - \alpha_C x_C \qquad [3]$$

$$\dot{x}_E = \beta_E x_E - \alpha_E x_E \qquad [4]$$

The death rates (α_C, α_E) have been modified to reflect the HAN3DY model. Since power per capita is a vital component which is an indicator of the development of society, the death rates from version2 of HAN3DY were adjusted to accommodate power. Also this equation, addresses the previous versions drawback of population surviving on renewable flows alone. As per version3 of HAN3DY model, decline below a threshold in per capita wealth (physical stocks), capital or power can cause the death rates to increase. Equations as per [69] and [70]

$$\alpha_C = \alpha_m + \max\left(0, 1 - \frac{C_C}{sx_C}, 1 - \frac{K_R + K_{NR}}{K_{TH}}, 1 - \frac{(P)}{P_{\_th}}\right) (\alpha_M - \alpha_m) \qquad [69]$$

$$\alpha_E = \alpha_m + \max\left(0, 1 - \frac{C_E}{sx_E}, 1 - \frac{(K_R + K_{NR})\kappa}{K_{TH}}, 1 - \frac{(P)\kappa}{P_{\_th}}\right) (\alpha_M - \alpha_m) \qquad [70]$$

The change in death rates vary from α_m (minimum death rate) to α_M (maximum death rate), in line with Figure 4.3 based on the minimum of the $\frac{(w_R + w_{NR})}{w_{TH}}$, $\frac{K_R + K_{NR}}{K_{TH}}$, $\frac{(P)}{P_{TH}}$ factors.

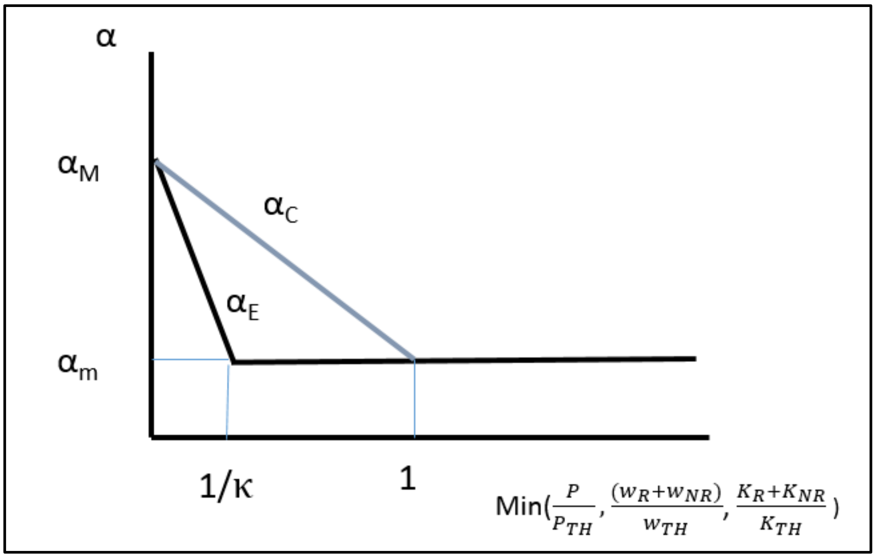


Figure 4.3: Death rates for elites are commoners as a function of wealth, capital and power.

The casual loop diagrams (see Figures 4.4 - 4.8), illustrate the links in version3 of HAN3DY discussed in Chapter 4. Here, the full model is represented in five Figures, and each represent a specific functional area of the model.

Figure 4.4: Casual loop diagram illustrating regenerative nature.

Figure 4.5: Casual loop diagram illustrating non-regenerative nature.

Figure 4.6: Casual loop diagram illustrating renewable flow nature.

Figure 4.7: Casual loop diagram illustrating power.

Figure 4.8: Casual loop diagram illustrating population.

In these Figures, Stocks are shown in boxes, and parameters in text.

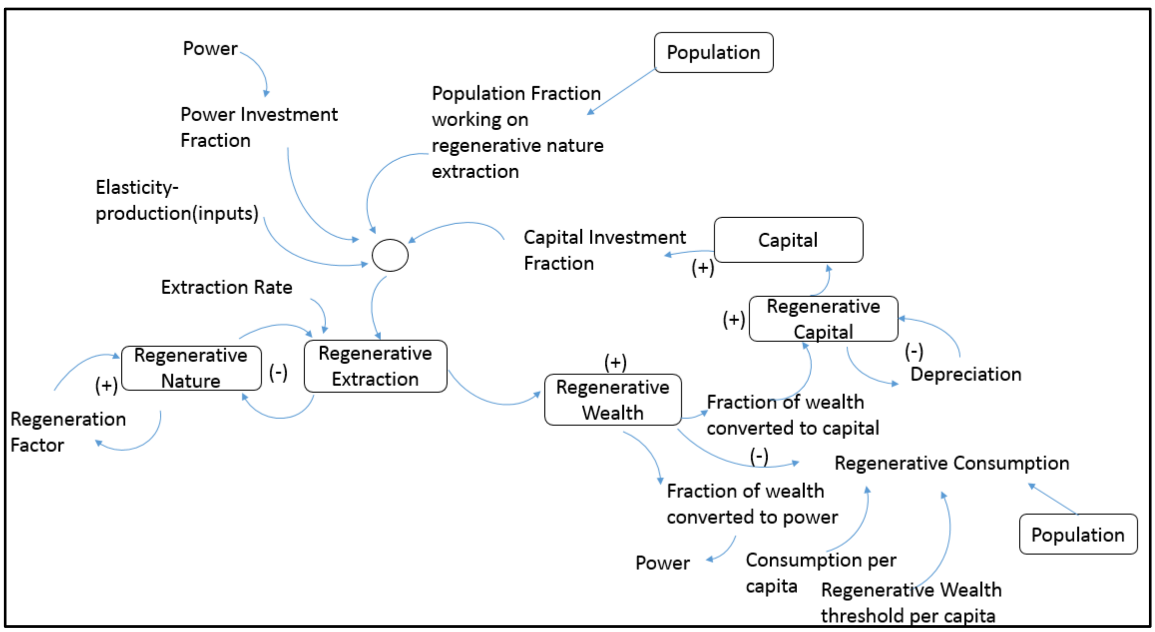


Figure 4.4: The casual loop diagram of HAN3DY model (v3), representing regenerative nature.

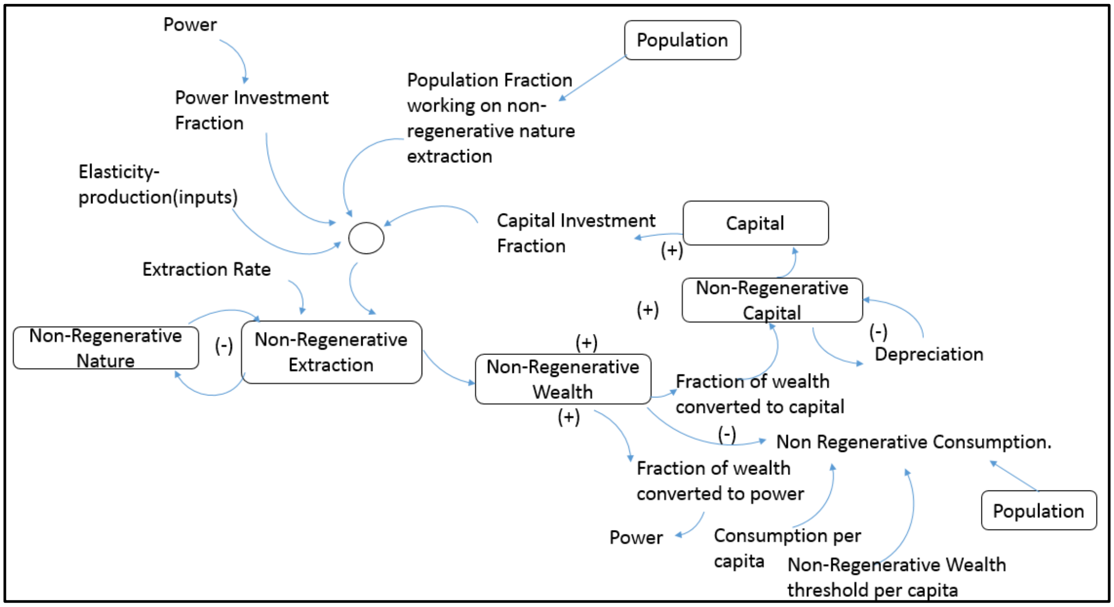


Figure 4.5: The casual loop diagram of HAN3DY model (v3), representing non-regenerative nature.

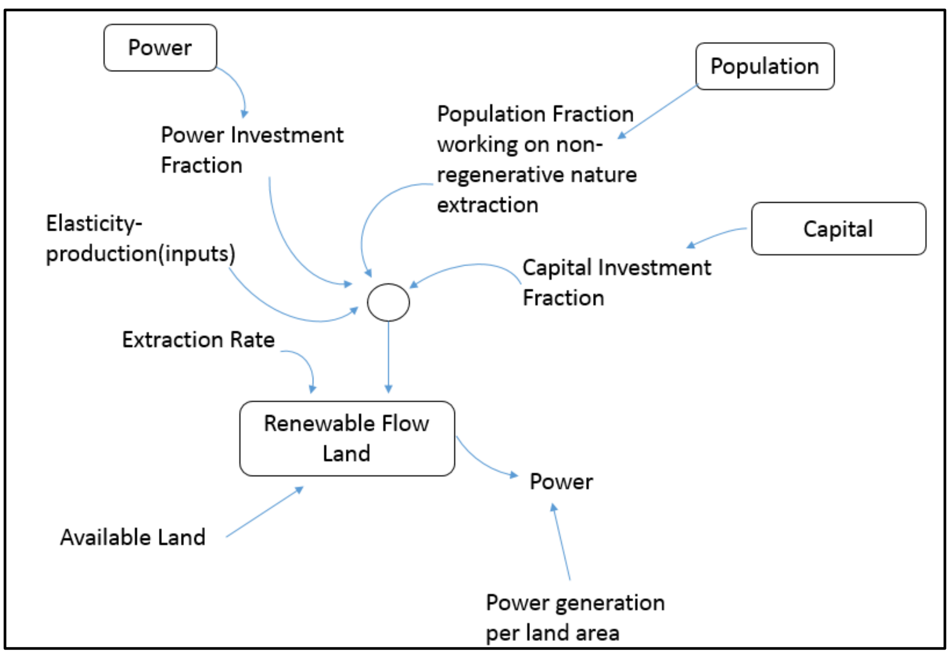


Figure 4.6: The casual loop diagram of HAN3DY model (v3), representing renewable flow nature.

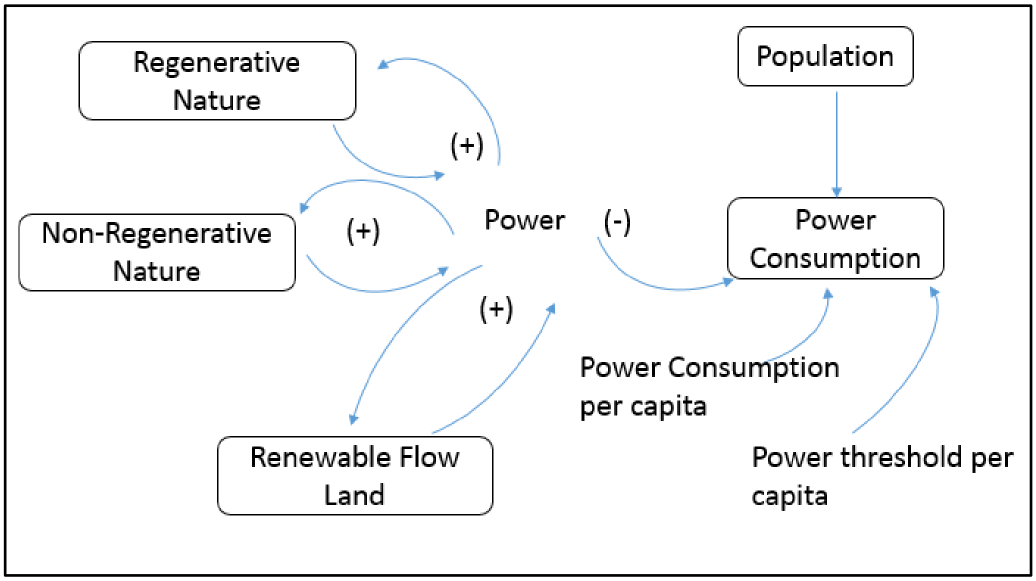


Figure 4.7: The casual loop diagram of HAN3DY model (v3), representing power.

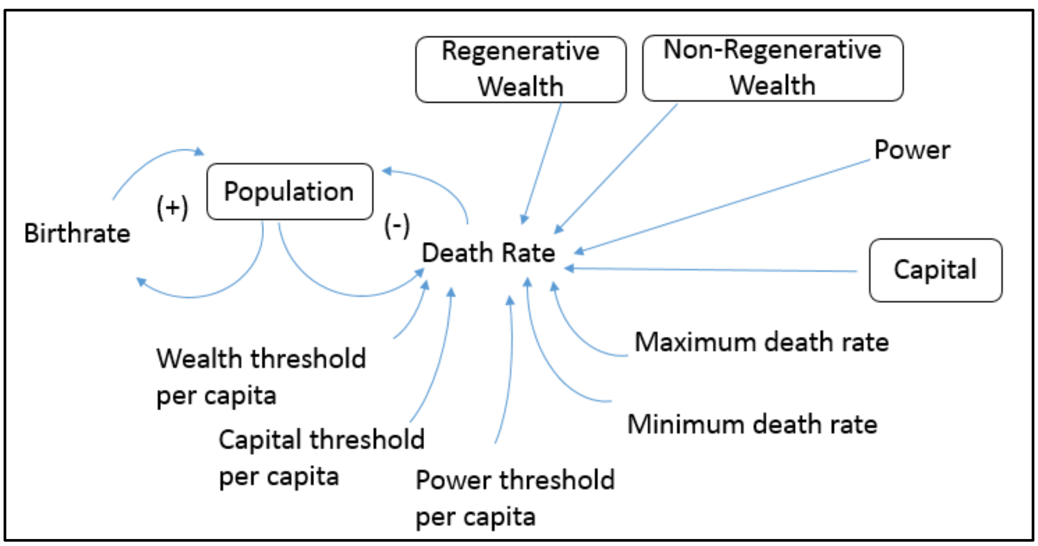


Figure 4.8: The casual loop diagram of HAN3DY model (v3), representing population.

Figure 4.9, is the stock flow diagram of HAN3DY (v3) model.

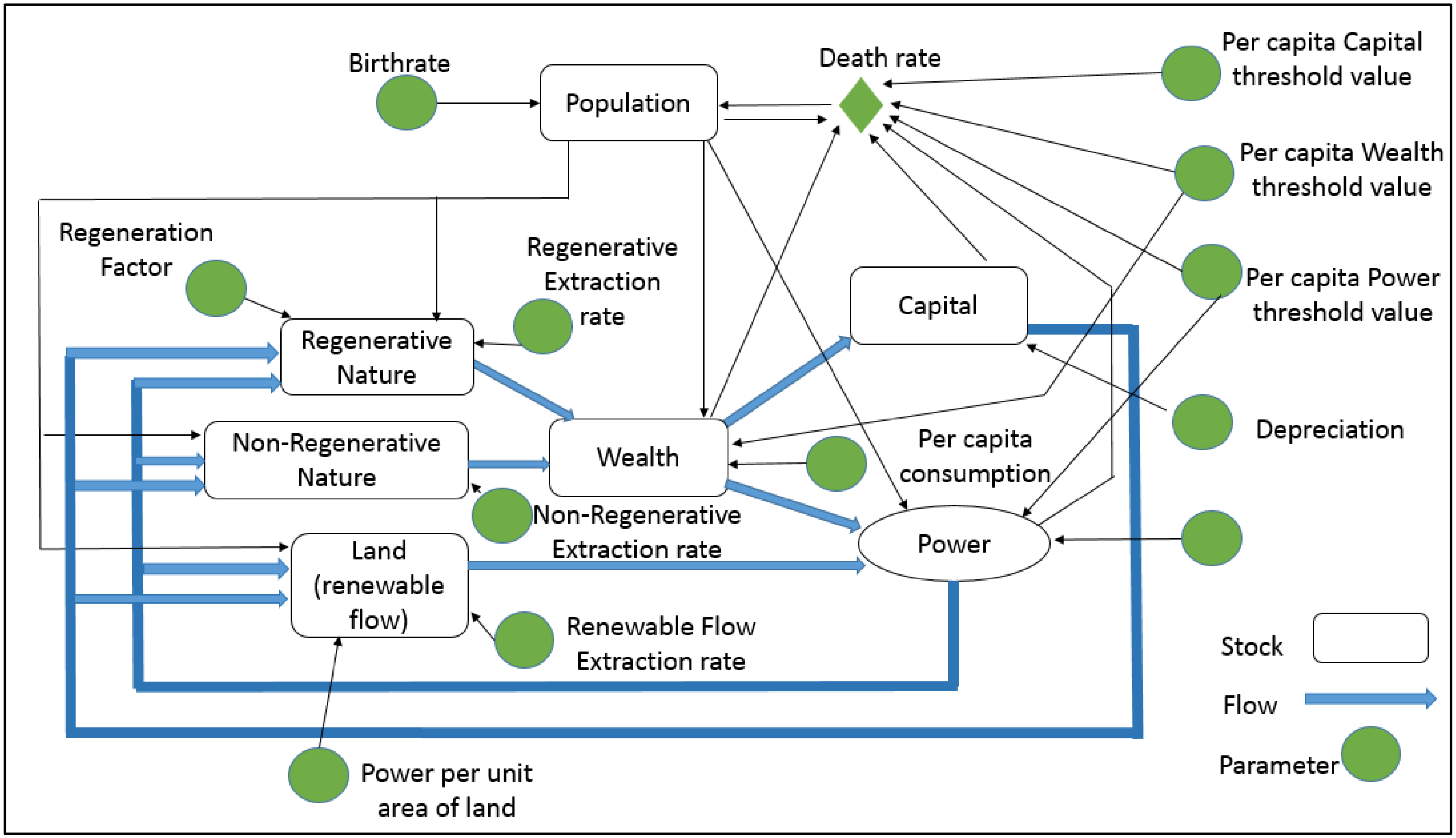


Figure 4.9: The stock-flow diagram of HAN3DY model (Version3).

4.3. Simulation Results of HAN3DY version3

The simulations are done for an egalitarian society with, $x_E = 0$. The weighting factors between the three input parameters in the production function are considered as equal: $\delta_{1,\mathrm{R}} = \delta_{1,\mathrm{NR}} = \delta_{1,\mathrm{RF}} = 0.5$, $\delta_{2,\mathrm{R}} = \delta_{2,\mathrm{NR}} = \delta_{2,\mathrm{RF}} = 0.66$. The elasticity of substitution factors used in the production function are as per published data on Germany ($\sigma_1 = 0.7931, \sigma_2 = 0.6980$) (Kemfert, 2000).

Simulation 4.3.1- The initial simulation is to understand the system dynamics with varying fractions of accumulated wealth converted towards power generation. Each nature extraction rate is maintained at the same value, $\boldsymbol{\delta}_{\mathbf{R}} = \boldsymbol{\delta}_{\mathbf{NR}} = \boldsymbol{\delta}_{\mathbf{RF}} = 6x10^{-3}$,$\mathrm{y}_{\mathrm{NR}} = 100, \lambda_F = 100, \mathrm{y}_{\mathrm{R}} = 100, x_{c,R} = 0.33x_C, x_{c,NR} = 0.33x_C$, $x_{c,RF} = 0.33x_C$,$w_{R(initial)} = 10$,$w_{NR(initial)} = 10$, $\mu_{NR} = \mu_R = 7.5\%$, (other parameters as per Appendix 1,Table 3).

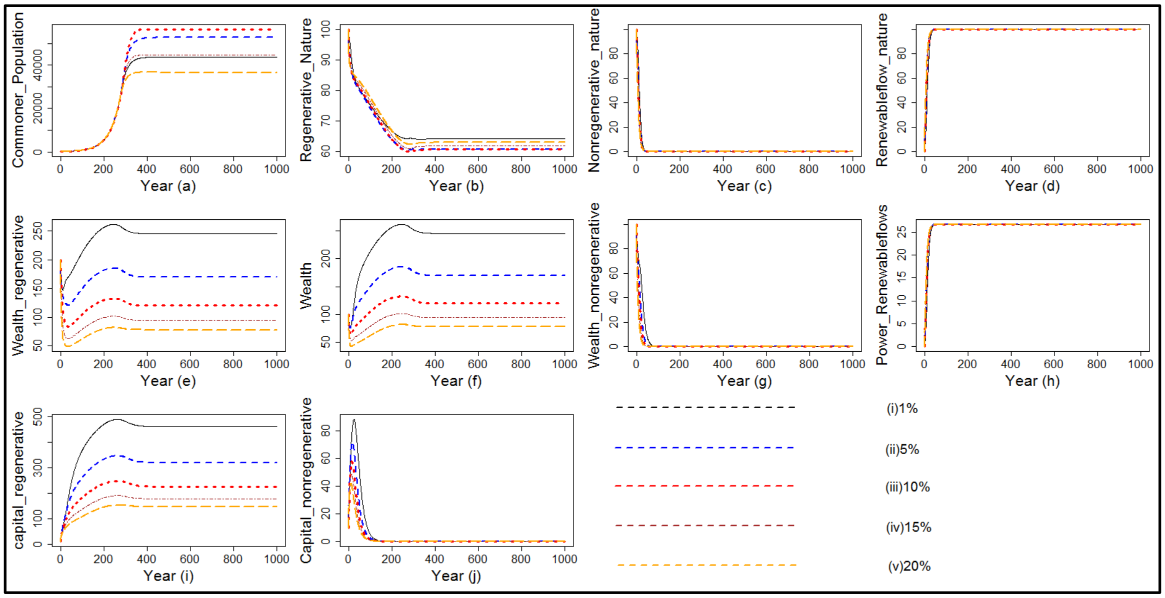


Figure 4.10: (a): Commoner Population, (b): Renewable nature, (c): Non-regenerative Nature, (d): Renewable Flow nature, (e): Wealth Renewable, (f) Wealth, (g): Wealth Non Regenerative, (h): Power Renewable Flows, (i) Capital Renewable resource, (j): Capital Nonrenewable resource. Simulations of HAN3DY v3, when varying levels of power being generated from regenerative and non-regenerative stocks. Simulation scenarios i) $\eta_R = \eta_{NR} = 1\%$, $ii)\eta_R = \eta_{NR} = 5\%$, $iii)\eta_R = \eta_{NR} = 10\%$, iv) $\eta_R = \eta_{NR} = 15\%, v)\ \eta_R = \eta_{NR} = 20\%$.

Observations- Simulation 4.3.1

It is observed that the maximum carrying capacity of population is achieved when the accumulated wealth fraction converted towards power is around 10% (or between 5% and 15%). When conversion levels are at the lowest 1%, the accumulated wealth is the highest, yet the lack of power in the system draws down the steady state population level. When the conversion level is high at 20%, the equilibrium population level is lower than at all other scenarios in Figure 4.10. When allocating 20% of wealth to power input for extraction there exists sufficient power for extraction, but the accumulated wealth is low which causes the population levels to keep a relatively low value. Hence it is seen, as per the parameters used in this simulation, the ideal fraction of wealth being converted to power in order to increase the population carrying capacity lies around the 10% mark for each of regenerative and non-regenerative wealth.

Simulation 4.3.2- This simulation is to understand the system dynamics with varying extraction rates for regenerative nature. As per the earlier assessment of conversion levels towards power generation. $\eta_R = \eta_{NR} = 10\%$, $\mu_R = \mu_{NR} = 7.5\%$. $\boldsymbol{\delta}_{\mathbf{NR}} = \boldsymbol{\delta}_{\mathbf{RF}} = 6x10^{-3}$, $y_{NR} = 100, \lambda_F = 100, y_R = 100, x_{c,R} = 0.33x_C$, $x_{c,NR} = 0.33x_C$, $x_{c,RF} = 0.33x_C$, $w_{R(initial)} = 10$, $w_{NR(initial)} = 10$ (other parameters as per Appendix 1,Table 3).

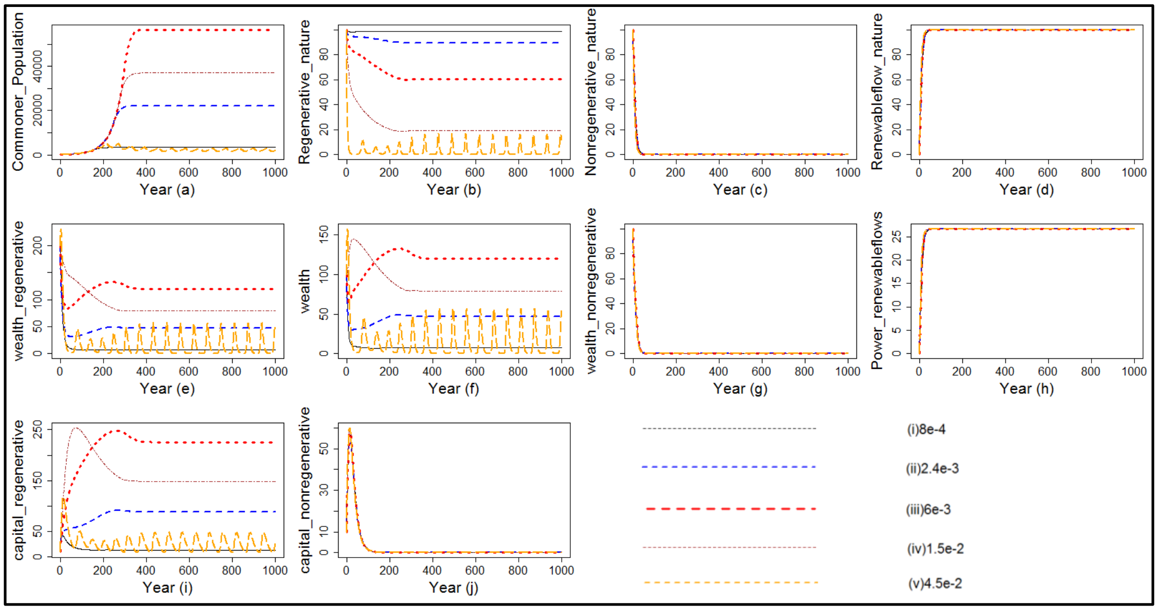


Figure 4.11: (a): Commoner Population, (b): Renewable nature, (c): Non-regenerative Nature, (d): Renewable Flow nature, (e): Wealth Renewable, (f) Wealth, (g): Wealth Non Regenerative, (h): Power Renewable Flows, (i) Capital Renewable resource, (j): Capital Nonrenewable resource. Simulations of HAN3DY v3, when extraction rates of non-regenerative and renewable flow resources held at a constant value $\delta_{NR} = \delta_{RF} = 6x10^{-3}(\text{person}^{-1} \bullet \text{time}^{-1})$6x10^{-3} , while varying extraction levels of regenerative stocks ($\boldsymbol{\delta}_{\mathbf{RF}}(\text{person}^{-1} \bullet \text{time}^{-1})$. Simulation scenarios (i) 8x10^{-4}, ii) 2.4x10^{-3}, iii) 6x10^{-3}, iv) 1.5x10^{-2}, v) 4.5x10^{-2}.

Observations-simulation 4.3.2

As per the simulation results in Figure 4.11 it is seen that the equilibrium population is the highest at extraction rate of 6x10^{-3} for regenerative resources. There are lower and higher extraction rates that results in lower population carrying capacity than the maximum. The maximum carrying capacity is achieved at an intermediate extraction rate of regenerative nature. In the instance when extraction is very low (8x10^{-4}, in scenario (i) of Figure 4.11) there is slow growth and the population converges to a low equilibrium value.

There are different limiting factors for population growth in Figure 4.11. Examples for some of the limiting factors are illustrated below. i) When extraction rate is low, the decline in growth in population is caused by the lack of capital in the system. Figure 4.12 illustrates the per capita capital value, and it's seen that the per capita capital value reduces down to the decline level, which causes the death rate to be equal to the birth rate. In this instance capital in the system is the governing factor on the population limit.

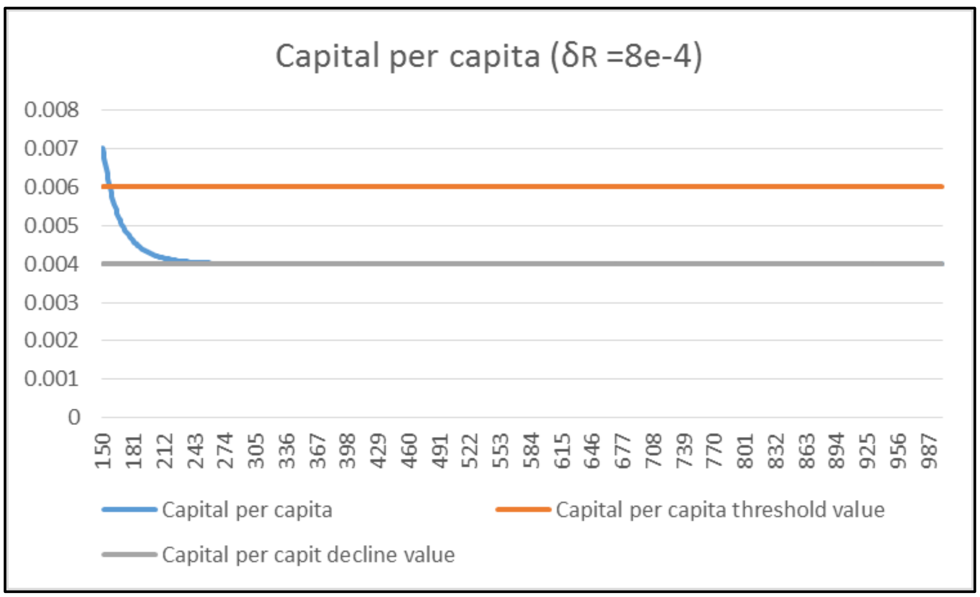


Figure 4.12: Capital per capita for simulation 4.3.2 (i) with refernece to per capita capital threshold value and per capita capital popualtion decline value.

ii) The popualtion's carrying capacity limitation for simulation 4.3.2 (iii) when extraction rate is $6x10^{-3}$ is due to the per capita power declining below threshold values towards the power per capita decline value. The per capita power reduces down to the decline level, thus causing the population to stabilize to an equilibrium value as illustrated in Figure 4.13.

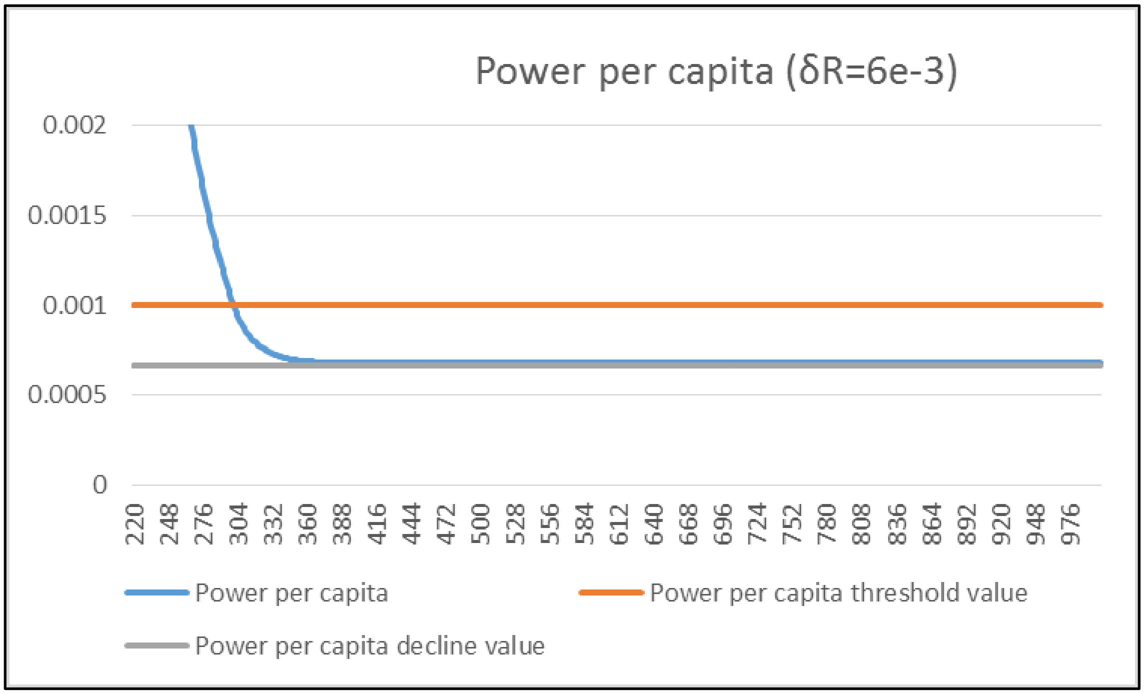


Figure 4.13: Power per capita for simulation 4.3.2 (iii) with refernece to per capita power threshold value and per capita power popualtion decline value.

iii) At the extreme extraction rate of $4.5x10^{-2}$ (simulation: 4.3.2 (v)) the cyclic oscillations in the population is due to the oscillatory behavior of the wealth per capita which is caused by cycles of high extraction and regeneration of regenerative nature . While this oscillation causes all other factors to oscillate, wealth per capita is the driving force behind the population characteristics. Figure 4.14 illustrates the change in wealth per capita that causes the oscillation.

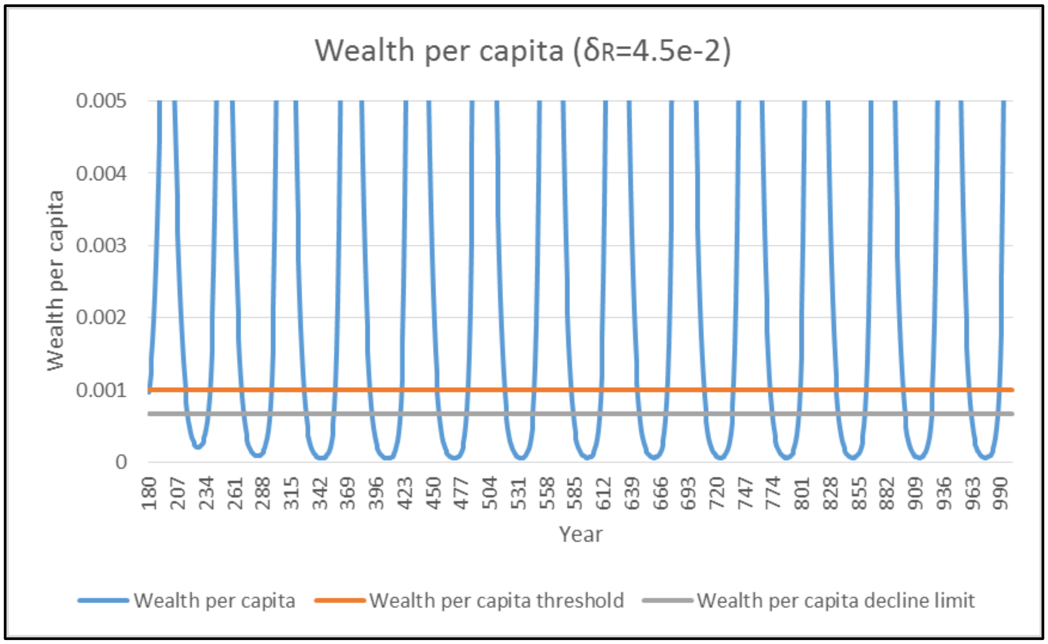


Figure 4.14: Wealth per capita for simulaiton 4.3.2 (v) with refernece to per capita wealth threshold value and per capita wealth popualtion decline value.

As the results suggest, all three economic indicators (wealth, capital and power) in the system have influence over the population levels.

Chapter 5: Scenario Simulations

5.1. Introduction

The previous chapters described in detail the systematic development, assumptions and equations that formulates the HAN3DY model. And this model seeks to represent the interactions between the human population, different kinds of nature resources, capital, wealth and power in a biophysical context.

While this chapter describes various situations based on thought experiments, it must be noted the results are based on the assumptions, and parameter values used in the model. This chapter uses version3 of HAN3DY model for the simulations. Furthermore, many tests were done to examine the sensitivity of the model in fine-tuning the parameters of HAN3DY.

5.2 Simulations

5.2.1. Three kinds of Nature

One of the HAN3DY model's main characteristic is the addition of two kinds of nature to the HANDY model. Hence, in this simulation the goal is to demonstrate the change in dynamics with the addition of the non-regenerative and the renewable flow resources to the regenerative resource. Three simulations are carried out, i) when only regenerative resources exist, ii) when regenerative and non-regenerative resources exist, iii) when regenerative, non-regenerative and renewable flow resources exist. The key parameters for each simulation are as in Table 3.1, and other parameters as per Appendix 1, Table 3.

Parameter	Simulation 1	Simulation 2	Simulation 3
Regenerative resource available ($y\lambda_{R}$)	100	100	100
Non regenerative resource available (λ_{NR})	0	100	100
Renewable flow resource available (λ_{RF})	0	0	100
Regenerative extraction rate (δ_{R})	$6x10^{-3}$	$6x10^{-3}$	$6x10^{-3}$
Non-regenerative extraction rate (δ_{NR})	0	$6x10^{-3}$	$6x10^{-3}$
Renewable flow extraction rate (δ_{RF})	0	0	$6x10^{-3}$
People working on regenerative nature extraction ($x_{c,R}$)	x_c	$x_c - x_{c,R}$	$x_c - x_{c,R} - x_{c,RF}$
People working on non-regenerative nature extraction ($x_{c,R}$)	0	$if(y_{NR} > 0), x_{c,NR} = 0.5x_C$ $else, x_{c,NR} = 0$	$if(y_{NR} > 0), x_{c,NR} = 0.5x_C$ $else, x_{c,NR} = 0$
People working on renewable flow nature extraction ($x_{c,RF}$)	0	0	$if(y_{RF} < 100)\ , x_{c,RF} = 0.33x_C$ $else,\ , x_{c,RF} = 0.1x_C$

Table 5.1 : Key parameters for simulations 5.2.1. (i), (ii) and (iii)

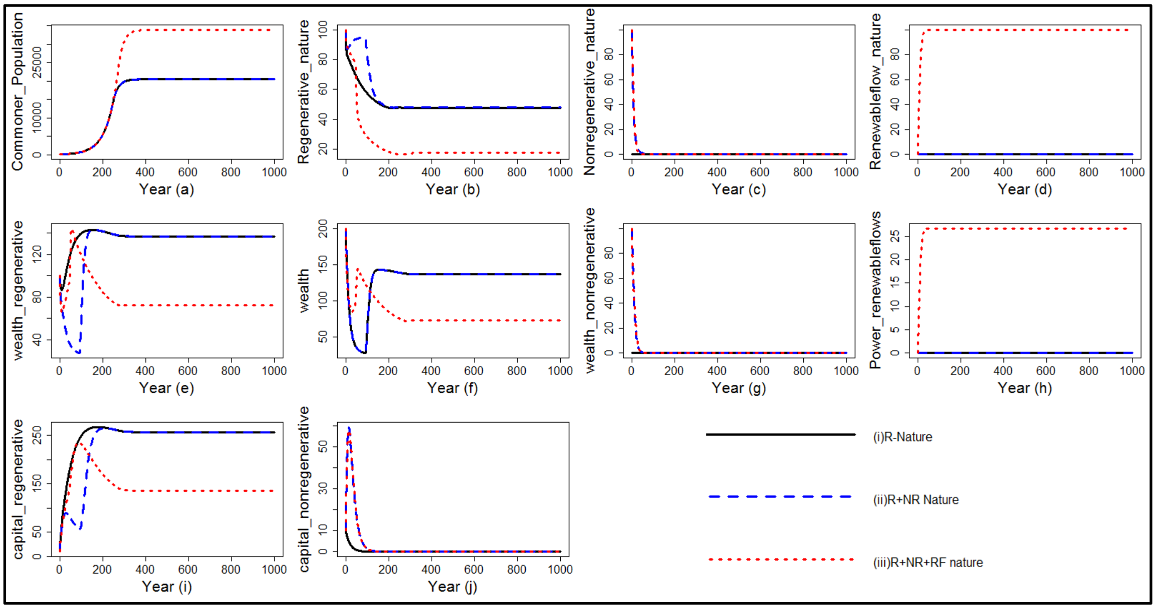


Figure 5.1: (a): Commoner Population, (b): Regenerative nature, (c): Non-regenerative Nature, (d): Renewable Flow nature, (e): Wealth Regenerative, (f) Wealth, (g): Wealth Non Regenerative, (h): Power Regenerative Flows, (i) Capital Regenerative resource, (j): Capital Nonrenewable resource. Simulations of HAN3DY v3, for simulation scenarios (i) only regenerative resources exist, ii) regenerative and non-regenerative resources exist, iii) regenerative, non-regenerative and renewable flow resources exist.

Observations-simulation 5.2.1

The dynamics of population when there exists only regenerative and when there exist both regenerative and non-regenerative resources are similar, for the above simulation with the used parameters (see Figure 5.1(a), simulations (i) and (ii)). This indicates that the wealth generated from non-regenerative resources compared to regenerative resources is insignificant at the current level of only 100 units of non-regenerative nature. Yet the addition of renewable flow resources help increase the population's carrying capacity. This increase in population carrying capacity is due to the power generated from the renewable flow resource being used to satisfy the populations' consumption (power) needs which reduces the dependency on regenerative nature. This contribution towards populations consumption needs by renewable flow nature thus helps increase the total population carrying capacity of the system.

The wealth contribution to the system by non-regenerative resources compared to regenerative resources is considerably low (see Figure 5.2), when the available non-regenerative nature, and regenerative nature are as per indicated by the limiting values($y_{NR} = 100, \lambda_R = 100$). This low contribution from non-regenerative nature has minimal impact on the population carrying capacity. As per the initial conditions, the varied contribution towards accumulated wealth brings to light the significance of the regenerative characteristic.

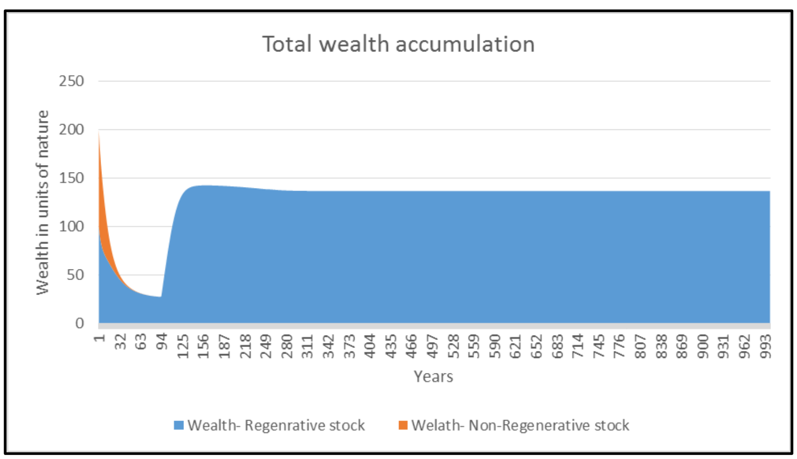


Figure 5.2: Contribution towards total wealth accumulation by regenerative stocks and non-regenerative stocks, for simulation 5.2.1(ii)

5.2.2. Quantity of Non-regenerative resources

As seen in the simulation of Section 5.2.1, the contribution towards wealth generation through non-regenerating nature is low. Hence in this simulation, the goal is vary the quantity of non-regenerative resources available, in understanding the contribution of it towards the system. The key parameters and assumptions used in this simulation are: $\lambda_{\mathrm{NR}} = 100{,}1000{,}10000$, $\lambda_F = 100, \lambda_{\mathrm{R}} = 100, \delta_R = 6\mathrm{x}10^{-4}, \delta_{NR} = \delta_{RF} = 6\mathrm{x}10^{-4}\eta_R = \eta_{NR} = 10\%, \mu_R = \mu_{NR} = 7.5\%,\ x_{c,R} = x_c - x_{c,NR} - x_{c,RF}, x_{c,NR} = if(\mathrm{y}_{\mathrm{NR}} > 0)0.33x_C\ else\ 0, x_{c,RF} = if(\mathrm{y}_{\mathrm{RF}} < 100)\ 0.33x_C\ else\ 0.1x_C$ (other parameters as per Appendix 1, Table 3).

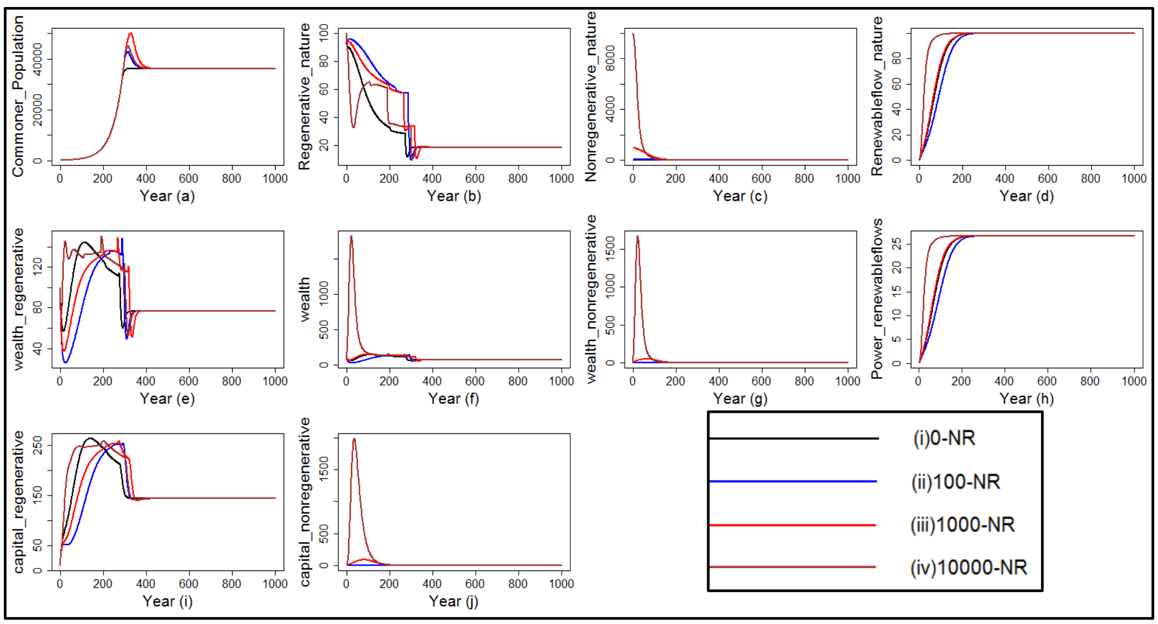
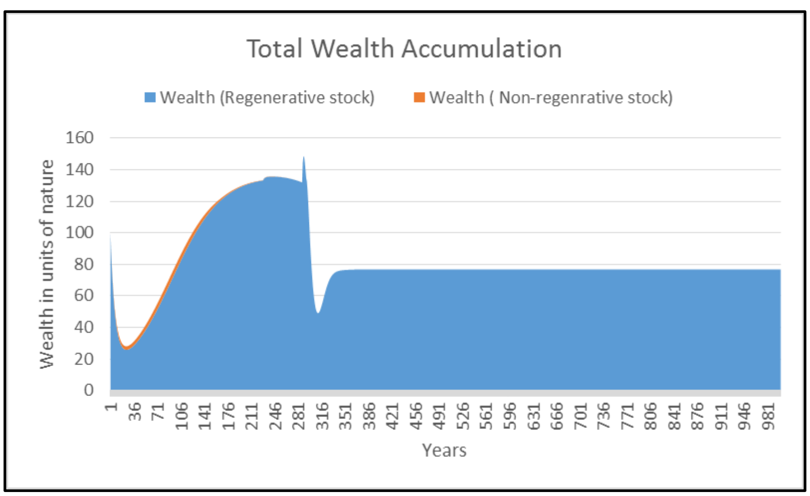


Figure 5.3: (a): Commoner Population, (b): Regenerative nature, (c): Non-regenerative Nature, (d): Renewable Flow nature, (e): Wealth Regenerative, (f) Wealth (g): Wealth Non Regenerative, (h): Power Regenerative Flows, (i) Capital Regenerative resource, (j): Capital Non-regenerative resource. Simulations of HAN3DY v3,when the available non-regenerative resource (λ_{NR}) quantity is being varied. Simulation scenarios (i) λ_{NR}=0 ii) λ_{NR}=100, iii) λ_{NR}=1000, iv) λ_{NR}=10000.

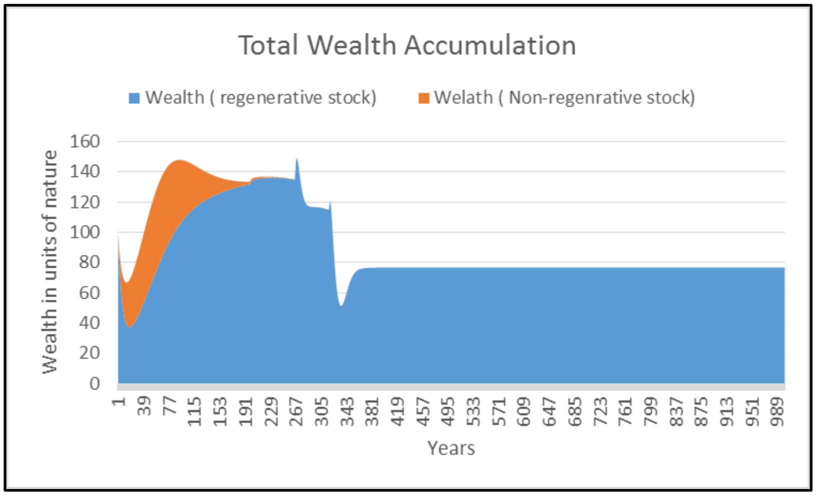
Observations-simulation 5.2.2

It is seen that equilibrium levels are achieved only after all non-regenerative resources have been exhausted and all renewable flow resources harnessed. The addition of the non-regenerative stock causes an initial spike in the population, which then recovers back to the equilibrium state after the non-regenerative resources, runs out. In this particular simulation the wealth being generated from regenerative resource alone is sufficient to keep the population growth at a maximum level initially. It is this that causes the population levels to grow at the same speed. Yet when wealth being generated from regenerative resources alone is not sufficient to keep growth rates at the maximum level, the addition of non-regenerative resources helps speed up the growth rate.

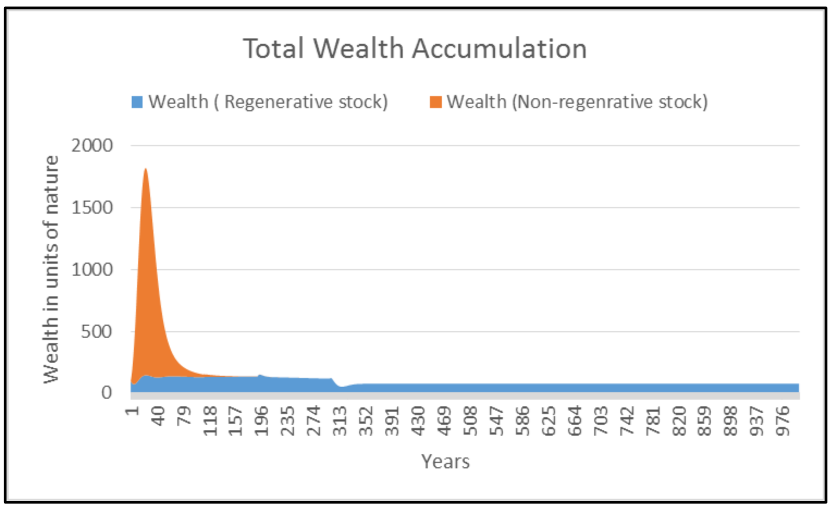
The contributions of non-regenerative and regenerative stocks towards wealth generation for the above simulation instances are illustrated in Figure 5.4.(a),(b) and (c).



(a)



(b)



(c)

Figure 5.4: Contribution towards total wealth accumulation by regenerative stocks and non-regenerative stocks, for simulation 5.2.2 when maximum available Non-regenerative stock varies. Simulation scenarios (a) λ_{NR}= 100, (b) λ_{NR}= 1000 (c) λ_{NR}= 10000.

5.2.3 Quantity of Renewable Flow Resources

This simulation was done in order to understand the effect to the system due to varying quantities of renewable flow nature (land) availability. Key parameters and assumptions: $\lambda_{\mathrm{NR}} = 100, \lambda_{RF} = 50{,}100{,}150, \lambda_{\mathrm{R}} = 100, \delta_R = 6\mathrm{x}10^{-3}, \delta_{NR} = \delta_{RF} = 6\mathrm{x}10^{-4}\eta_R = \eta_{NR} = 10\%, \mu_R = \mu_{NR} = 7.5\%,\ x_{c,R} = x_c - x_{c,NR} - x_{c,RF}, if(\mathrm{y_{NR}} > 0)x_{c,NR} = 0.33x_C\ else x_{c,NR} =\ 0, if(\mathrm{y_{RF}} < 100)x_{c,RF} = 0.33x_C\ else x_{c,RF} =\ 0.1x_C$ (other parameters as per Appendix 1,Table 3)

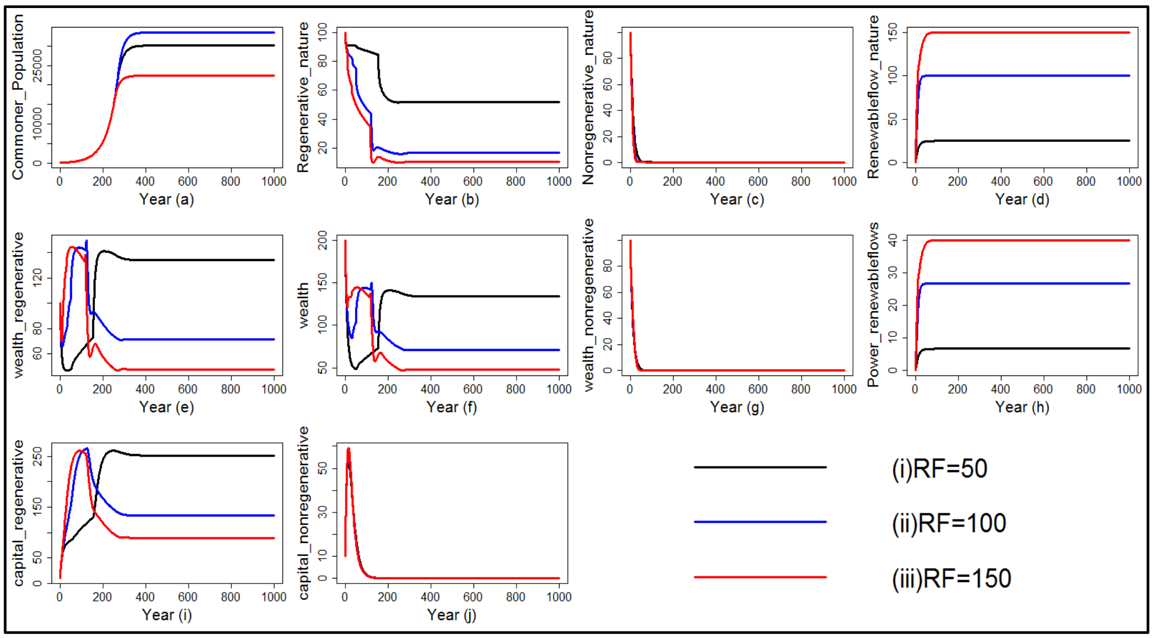


Figure 5.5: (a): Commoner Population, (b): Regenerative nature, (c): Non-regenerative Nature, (d): Renewable Flow nature, (e): Wealth Regenerative, (f) Wealth, (g): Wealth Non Regenerative, (h): Power Regenerative Flows, (i) Capital Regenerative resource, (j): Capital Nonrenewable resource. Simulations of HAN3DY v3, when renewable flow land (λ_{RF}) quantity is being varied. Simulation scenarios i) When, λ_{RF} =50, ii) When, λ_{RF} =100, iii) When, λ_{RF} =150).

Observations-simulation 5.2.3

As per the parameters used in the simulation, it is seen that initially there is an increase in the population carrying capacity with the increase in renewable flow land availability, yet further increases causes the population carrying capacity to reduce. The reduction in population carrying capacity from simulation 5.2.3 (ii) to (iii) is caused due to the decline in per capita capital. HAN3DY model is set up, such that additional power beyond population consumption is directed towards extraction processes. When available renewable flow resources are very high, this results in higher additional power beyond the populations' consumption requirement towards nature extractions. This high levels of power towards the extraction processes causes nature extraction to increase which eventually causes abrupt societal collapse due to, resources being exhausted. This could be interpreted as high incentive to extract nature causing a negative impact.

5.2.4 Wealth Accumulation

In a market driven economy, maximizing wealth is the ultimate business option. The following simulation was done in order to understand the system dynamics in trying to increase the accumulated wealth. Here multiple simulations are done for varying extraction rates. Key parameters used are $\lambda_{\mathrm{NR}} = 100, \lambda_F = 100, \lambda_{\mathrm{R}} = 100, \delta_{NR} = \delta_{RF} = 6\mathrm{x}10^{-2} \eta_R = \eta_{NR} = 10\%, \mu_R = \mu_{NR} = 7.5\%,$ $x_{c,R} = x_c - x_{c,NR} - x_{c,RF}, x_{c,NR} =$

$if(y_{NR} > 0)\ 0.33x_C\ else\ 0, x_{c,RF} = if(y_{RF} < 100) 0.33x_C\ else\ 0.1x_C$ (other parameters as per Appendix 1,Table 3).

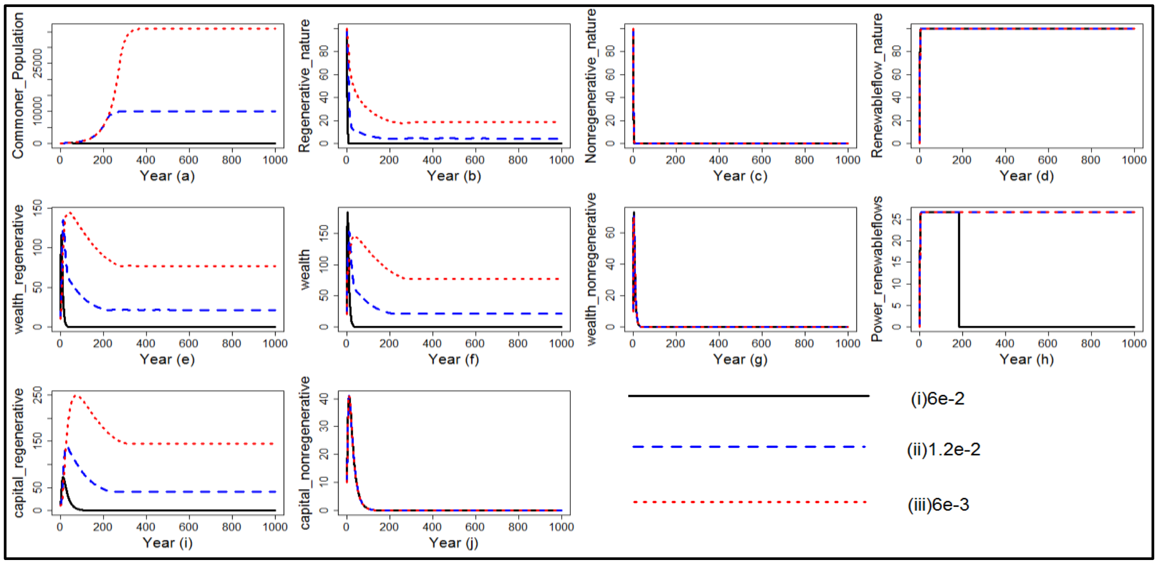


Figure 5.6: (a): Commoner Population, (b): Regenerative nature, (c): Non-regenerative Nature, (d): Renewable Flow nature, (e): Wealth Regenerative, (f) Wealth, (g): Wealth Non Regenerative, (h): Power Regenerative Flows, (i) Capital Regenerative resource, (j): Capital Nonrenewable resource. Simulations of HAN3DY v3, When regenerative extraction rate (δ_R ($\text{person}^{-1} \bullet \text{time}^{-1}$))is varied to increase the accumulation of wealth. Simulation scenarios i) $\delta_R = 6\text{x}10^{-2}$, $ii)$ $\delta_R = 1.2\text{x}10^{-2}$, iii)$\delta_R = 6\text{x}10^{-3}$

Observations-simulation 5.2.4

It is seen that accumulated wealth peaks when extraction rate of regenerative resource is at the highest $\delta_R = 6\text{x}10^{-2}$(see in Figure 5.7). Although the accumulated wealth is at the highest, it is achieved at the expense of all the nature available. Since extraction rates of non-regenerative and renewable flow nature are set at a high value, there is total contribution in terms of wealth and power from these resources at the point of peak. The exact point when accumulated wealth peak is when regenerating resource drops to zero. Non-regenerative resource runs out prior to regenerative resource being fully exhausted. This initial spike in wealth causes the population to grow, yet the growth is short lived since the system runs out of nature to generate wealth and power in sustaining the population. Hence, it is evident that high accumulated wealth and population sustainability do not go hand in hand.

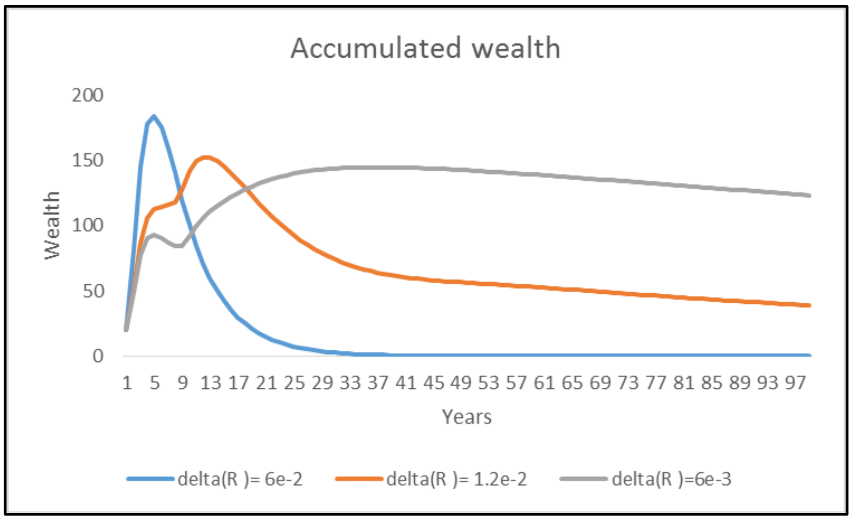


Figure 5.7 : Accumulated total wealth for simulation 5.2.4.

Chapter 6: Conclusion

All biophysical quantities growing within a finite space would either i) transition to an equilibrium position, ii) oscillate around an equilibrium state or iii) overshoot and decline (Meadows, D, et.al, 1972). The Human and three-Nature Dynamics model (HAN3DY) attempts to investigate behavior modes between human and nature through thought experiments. This allows for understanding of the dynamics of key components (capital, accumulated wealth, population, power, resources) with reference to key parameters (extraction rates, resource levels, etc.).

HAN3DY model is a frame work developed to investigate the likely characteristics of population and resources, in better understanding an energy transition. HAN3DY expands the capabilities of the HANDY model through the addition of key features while keeping it as simple as possible. This improved HAN3DY model consists of the three kinds of nature available (regenerative, non-regenerative and renewable flows) to mankind, for all its physical and energy needs. Also the power generation from the three kinds of resources, and the extraction processes involving key input parameters of labor, capital, and power, while considering the elasticity of substitution between them are some of the key features of the HAN3DY model. In addition to the above, there is disaggregation of human consumption as physical stock and power in HAN3DY and its feedback loops towards population growth have been reconfigured as detailed in Chapter 2. Each kind of nature, capital stocks and power generation being treated independently, helps better understand the rates of transition between sources as well as optimal solutions. Hence, this model could be used to understand the timeline for energy transitions such that disturbances in population levels could be minimized.

In conclusion, the thought experiments conducted in Chapters 2 and 3 revealed that the ultimate cause of societal collapse is the high extraction of resources. Additionally, the following features regarding the human-nature interactions were also revealed:

1) Sustainability of the population is not possible, if the system is solely dependent on non-regenerative resources, renewable flow resource or the combination of the two. Non-regenerative resources will continue to grow the population levels until all non-regenerative resources have been extracted, at which the population will start to decline. Since renewable flow resources can only generate power, and population is dependent on physical stocks for its existence, this resource on its own, cannot sustain a population.
2) Non zero equilibrium value of population (if conducive) will only be reached after all non-regenerative resources are exhausted and renewable flow resources harnessed, since regenerative nature is the only resource that could generate physical stock and power, which are vital for the existence of the population while regenerating its resource base. Equilibrium value is obtained when the rate of regeneration equals rate of extraction.

3) The wealth contribution to the system due to the regeneration phenomena of the regenerative nature resource is significant. This is since, regeneration is a continuous process through time, and can be clearly seen, when compared with non-regenerative resource.
4) Non-regenerative resources help increase the growth rate of the population, which could lead to a decline or an oscillation in population levels when this resource is fully depleted.
5) Renewable flow resource helps increase the carrying capacity of the population since the power generated from this resource, reduces the dependency on other resources. This eventually results in the increase in the population carrying capacity.

Steps for further development

HAN3DY model is an advancement of the HANDY model that helps better understand the dynamics between human population and different kinds of nature. Yet like all models, this could be further expanded to increase its capabilities. Some possible steps are,

1) Prioritizing generation of power, for example: generate power as per the requirement of the population's consumption needs and for the extraction processes only in line with the capital/power requirement.
2) Combining the HAN3DY model with an economic model.

In attempting to draw parallels with some key features gained from the HAN3DY model and the current world conditions, it is seen that currently the world is dependent heavily on non-regenerative resources for both physical (metals, materials) and power needs (fossil fuels). As per the understandings of the model when there exist high dependency on non-regenerative resources there is high growth in population, which could go beyond the population carrying capacity of the world . Unless there is gradual shift towards regenerative and renewable flow resources, an oscillatory behavior of population level or a drastic decline in the level of population would be inevitable with the non-regenerative resources fully depleting.

Appendix 1

Table 1 : Parameters vales used for HANDY and HAN3DY (V1) models

Parameter Symbol	Parameter	Typical Value
λ_R	Regenerative nature carrying capacity	100
λ_F	Renewable Flow Nature carrying capacity (Maximum available land)	100
δ_F	Extraction rate of renewable flow nature.	6.67e-6
δ_{NR}	Extraction rate of regenerative nature	6.67e-6
I	Extraction rate of renewable flow power per land area.	0.267
ρ	Threshold wealth per capita	5e-3
κ	Inequality Factor	1
S	Subsistence Salary per Capita	5e-4
α_M	Maximum Death rate	7e-2
α_m	Minimum Death Rate	1e-2
β	Birth Rate	3e-2
γ	Regeneration factor	0.1
$x_{c(initial)}$	Initial commoner population	100
θ	Electricity consumption as a fraction of total consumption.	0.5
ϑ_R	Percentage of commoners working in Regenerative Nature extraction	0.33
ϑ_{NR}	Percentage of commoners working in Non-Regenerative Nature extraction	0.33

Table 2: Parameters vales used for HAN3DY (V2) model

Parameter Symbol	Parameter	Typical Value
λ_R	Regenerative nature carrying capacity	100
λ_F	Renewable Flow Nature carrying capacity (Maximum available land)	100
δ_F	Extraction rate of renewable flow nature.	6.67e-6
δ_{NR}	Extraction rate of regenerative nature	6.67e-6
I	Extraction rate of renewable flow power per land area.	0.267
ρ	Threshold wealth per capita	5e-3
ρ_R	Threshold regenerative capital per capita	0.5e-3
ρ_{NR}	Threshold non-regenerative capital per capita	0.5e-3
ρ_{KR}	Threshold regenerative capital per capita	3e-3
ρ_{KNR}	Threshold non-regenerative capital per capita	3e-3
κ	Inequality Factor	1
S	Subsistence Salary per Capita	1e-4
S_R	Subsistence Salary per Capita-regenerative resources	0.5e-4
S_{NR}	Subsistence Salary per Capita-non regenerative resources	0.5e-4
α_M	Maximum Death rate	7e-2
α_m	Minimum Death Rate	1e-2
β	Birth Rate	3e-2
γ	Regeneration factor	0.1
$x_{c(initial)}$	Initial commoner population	100
θ	Electricity consumption as a fraction of total consumption.	0.5
ϑ_R	Percentage of commoners working in Regenerative Nature extraction	0.33
ϑ_{NR}	Percentage of commoners working in Non-Regenerative Nature extraction	0.33

$\emptyset_{R}$	Regenerative Capital Depreciation rate	0.05
$\emptyset_{NR}$	Non-Regenerative Capital Depreciation rate	0.05
$\boldsymbol{\varepsilon}$	Wealth/capital ratio	0.8
$\boldsymbol{\delta}_{\mathbf{1,R/NR/RF}}$	Weighting factors for regenerating/non-regenerating/renewable flow nature extraction	0.5
$\boldsymbol{\delta}_{\mathbf{2,R/NR/RF}}$	Weighting factors for regenerating/non-regenerating/renewable flow nature extraction	0.66
$\boldsymbol{\rho}_{\mathbf{1}}$	Elasticity of Substitution between capital and labor	0.2608
$\boldsymbol{\rho}_{\mathbf{2}}$	Elasticity of Substitution between capital and labor	0.4325
$\boldsymbol{f}_{\boldsymbol{R\to R}}$	Percentage of regenerative capital towards regeneration resource extraction	0.33
$\boldsymbol{f}_{\boldsymbol{NR\to R}}$	Percentage of non-regenerative capital towards regeneration resource extraction	0.33
$\boldsymbol{f}_{\boldsymbol{R\to NR}}$	Percentage of regenerative capital towards non-regeneration resource extraction	0.33
$\boldsymbol{f}_{\boldsymbol{NR\to NR}}$	Percentage of non-regenerative capital towards non-regeneration resource extraction	0.33

Table 3: Parameters vales used for HAN3DY (V3) model

Parameter Symbol	**Endogenous Variables(EV)/ Parameters(P)**	**Typical Value**
$\boldsymbol{\lambda}_{\mathbf{R}}$	Regenerative nature carrying capacity	100
$\boldsymbol{\lambda}_{\mathbf{F}}$	Renewable Flow Nature carrying capacity (Maximum available land)	100
$\boldsymbol{\delta}_{\mathbf{F}}$	Extraction rate of renewable flow nature.	6.67e-6
$\boldsymbol{\delta}_{\mathbf{NR}}$	Extraction rate of regenerative nature	6.67e-6
I	Extraction rate of renewable flow power per land area.	0.267
$\boldsymbol{\rho}$	Threshold wealth per capita	5e-3
ρ_{R}	Threshold regenerative capital per capita	0.5e-3
ρ_{NR}	Threshold non-regenerative capital per capita	0.5e-3

ρ_E	Threshold non-regenerative power per capita	1e-3
ρ_{KR}	Threshold regenerative capital per capita	3e-3
ρ_{KNR}	Threshold non-regenerative capital per capita	3e-3
κ	Inequality Factor	1
S	Subsistence Salary per Capita	5e-4
$\mathbf{S_R}$	Subsistence Salary per Capita-regenerative resources	0.5e-4
$\mathbf{S_{NR}}$	Subsistence Salary per Capita-non regenerative resources	0.5e-4
$\mathbf{S_E}$	Subsistence Salary per Capita-power	1e-4
$\boldsymbol{\alpha_M}$	Maximum Death rate	7e-2
$\boldsymbol{\alpha_m}$	Minimum Death Rate	1e-2
β	Birth Rate	3e-2
γ	Regeneration factor	0.1
$\mathbf{x_{c(initial)}}$	Initial commoner population	100
θ	Electricity consumption as a fraction of total consumption.	0.5
ϑ_R	Percentage of commoners working in Regenerative Nature extraction	0.33
ϑ_{NR}	Percentage of commoners working in Non-Regenerative Nature extraction	0.33
$\emptyset_R$	Regenerative Capital Depreciation rate	0.05
$\emptyset_{NR}$	Non-Regenerative Capital Depreciation rate	0.05
$\boldsymbol{\varepsilon}$	Wealth/capital ratio	0.8
$\boldsymbol{\delta_{1,R/NR/RF}}$	Weighting factors for regenerating/non-regenerating/renewable flow nature extraction	0.5
$\boldsymbol{\delta_{2,R/NR/RF}}$	Weighting factors for regenerating/non-regenerating/renewable flow nature extraction	0.66
$\boldsymbol{\rho_1}$	Elasticity of Substitution between capital and labor	0.2608
$\boldsymbol{\rho_2}$	Elasticity of Substitution between capital and labor	0.4325
$\boldsymbol{f_{R \rightarrow R}}$	Percentage of regenerative capital towards regeneration resource extraction	0.33
$\boldsymbol{f_{NR \rightarrow R}}$	Percentage of non-regenerative capital towards regeneration resource extraction	0.33

$\boldsymbol{f_{R \to NR}}$	Percentage of regenerative capital towards non-regeneration resource extraction	0.33
$\boldsymbol{f_{NR \to NR}}$	Percentage of non-regenerative capital towards non-regeneration resource extraction	0.33

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