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Macroscale Modeling Linking Energy and Debt: A Missing Linkage

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Macroscale Modeling Energy and Debt: A Missing Linkage

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Abstract

Macroscale Modeling Linking Energy and Debt: A Missing Linkage

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What if we realized that the fundamental economic framework of models that are meant to guide a low-carbon energy transition prevents them from actually answering the question they are supposed to answer? Instead of assuming a series of energy investments, and then estimating the economic impacts of those choices, they actually do the exact opposite. They assume economic growth and then make a series of investments to meet emissions targets without actually factoring in how the energy systems themselves feedback to economic growth. The research here would be to try to understand how energy and resource extraction are linked with long-term economic outcomes, specifically addressing the idea of accumulation of debt in the economy. Many economic models implicitly assume that energy resources are not constraints on the economy. These energy-related constraints have to be introduced if we are to effectively understand long-term debt and natural resource interactions. Same is also true with various biophysical models which do not consider economic parameters like debt, employment and wages etc. while modeling population growth and resources in the system.

The research objective is to develop a consistently merged model combining both a biophysical and an economic model to describe the industrial transition to the contemporary macroeconomic state. The research approach would be to integrate macro-scale system dynamics models of money, debt, and employment (specifically the Goodwin and Minsky models of (Keen, 1995 & Keen, 2013)) with system dynamics models of biophysical quantities (specifically population and natural resources such as in (Meadows et al., 1972, Meadows et al., 1974, Motesharrei et al., 2014)). The proposed research

concept is critical to link biophysical modeling concepts with those economic models that specifically include the link of debt to employment and economic growth.

This type of modeling is anticipated to help answer important questions for a low-carbon transition, for example, how does the rate of investment in "energy" feedback to growth of population, economic output, and debt; and how does the capital structure (e.g. fixed costs vs. variable costs) of fossil and renewable energy systems relate to, and affect, economic outcomes.

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1.1. Research Question

To understand how energy and resource extraction are linked with long-term economic outcomes, including the accumulation of debt in the economy.

1.2. Problem Statement

Many economic models implicitly assume that energy resources are not constraints on the economy. These energy-related constraints have to be introduced if we are to effectively understand long-term debt and natural resource interactions. This is also true with various biophysical models that do not consider economic parameters like Debt, Employment, and Wages. while modeling population growth and resource extractions.

1.3. Research Objective

This objective seeks a consistent biophysical and economic framework to describe the industrial transition to our contemporary macroeconomic state. Here the research seeks to integrate macro-scale system dynamics models of money, debt, and employment (specifically the Goodwin and Minsky models of (Keen, 1995 and Keen, 2013)) with system dynamics models of biophysical quantities (specifically population and natural resources such as in (Meadows et al., 1972, Meadows et al., 1974, Motesharrei et al., 2014)). In other words, there are models of each separately, but they have not been combined to fundamentally link the biophysical world to monetary frameworks. The proposed research concept is critical to link biophysical modeling concepts with those economic models that specifically include the link of debt-based finance to employment and economic growth.

2. Chapter 2: Previous Research

2.1. Literature Review

2.1.1. Origin of Humans[1,2]

Homininans are assumed to be living on this planet for over 5 million years, probably when some apelike creatures in Africa began to walk habitually on two legs. They were flaking crude stone tools by 2.5 million years ago. Then some of them spread from Africa into Asia and Europe after 2 million years ago. The modern human (*Homo sapiens*, the only extent members of the *Hominina Tribe*, *the Homininans*) is supposedly had to evolve from ancestors who had remained in Africa. They, too, moved out of Africa and eventually replaced non-modern human species, notably the Neanderthals in Europe and parts of Asia, and Homo erectus, typified by Java Man and Peking Man fossils in the far East (Figure 1). An increase in population and competition and the ability to shape sophisticated tools, hunt big game, and build permanent shelter may have spurred the first wave of migration of *Homo sapiens* from Africa to the Middle East about 100,000 years ago[3]. From there people slowly made their way into Central Asia and onward. A new push into Southeast Asia occurred about 75,000 years ago, and as ice age cooled the earth and water were concentrated in massive glaciers, the earth's oceans receded and exposed land bridges between continents. Taking a fragment of the islands of Indonesia and New Guinea. By 60,000 BC some groups also crossed from New Guinea to Siberia and Alaska allowed humans to cross into Americas around 16,000 BC. By 11,000 BC. The human had reached the southernmost tip of South America.

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1 http://www.nytimes.com/2002/02/26/science/when-humans-became-human.html

2 http://www.lanbob.com/lanbob/H-History/HH-02-Societies250KBC-500.html

3 http://www.lwrw.org/Part2.htm

Human Migration Map[4]

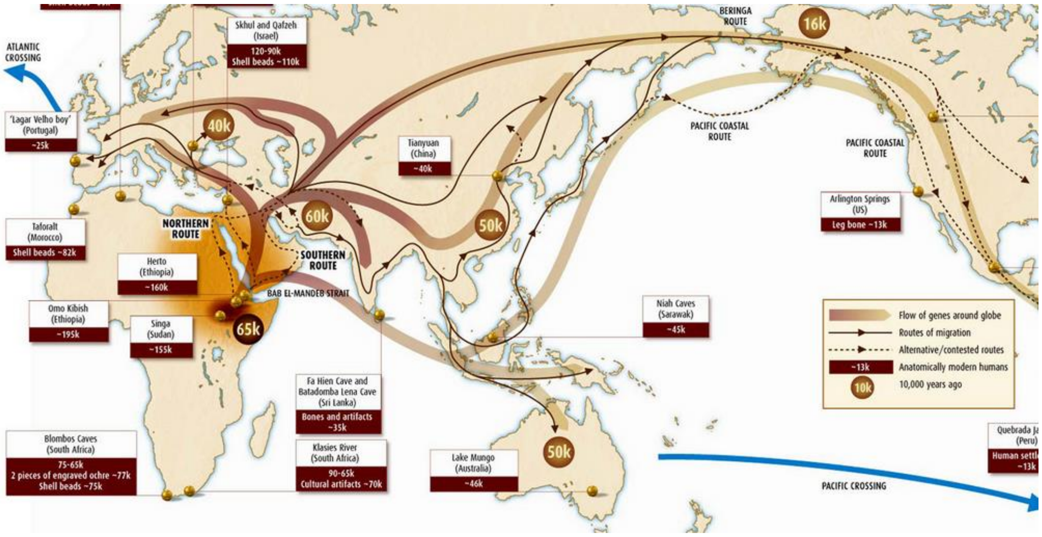

Figure 1. The northern route was taken by archaic *Homo sapiens* from East Africa via the Sinai into Israel 120,000 years ago and it also shows the Southern Route taken by *Homo sapiens* from East Africa via the Bab-al-Mandab strait into Yemen 70,000 years ago.

Humans owe their phenomenal ability to alter the world for good or ill to a process of evolution that began in Africa more than four million years ago, with the emergence of the first hominids: primates with the ability to walk upright. Early hominids stood only three or four feet tall. On average and had brains roughly one-third the size of the modem human brain, which limited their capacity to reason or speak. But their upright posture and opposable thumbs (used to grip objects between fingers and thumb) allowed them to gather and carry food and process it using simple tools.

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4 http://www.lwrw.org/Part2.htm

Over time, other species of hominids evolved that possessed larger brains and the ability to fully articulate their thoughts, craft ingenious tools and weapons, and hunt collectively. Homo sapiens, or modern humans, emerged in Africa perhaps 250,000 years ago, and they had a talent for adapting to changing circumstances that allowed them to occupy much of the planet (Trinkaus 2005) and (Antón and Swisher III 2005). By clothing themselves in animal hide and living in caves warmed by fires, they survived winters in northern latitudes during the most recent phase of the Ice Age, which came to an end around 12,000 years ago. That glaciation lowered sea levels and enabled people to walk from Siberia to North America and reach Australia and other previously inaccessible land masses.

During the Ice Age, humans gathered wild grains and other plants, but they owed their survival and success largely to hunting. They became so, skilled were in hunting in groups and killing larger animals that they probably contributed to the extinction of such species as the mammoth and mastodon.[5] Hunters paid tribute to the animals they stalked in cave paintings that may have been intended to honor the spirit of those creatures so they would offer up their bounty. From early times, the destructive power of humans as predators was linked to their creative power as artists and inventors.

The warming of the planet that began around 10,000 BC forced humans to adapt, and they did so with great ingenuity. Many of the larger animal's people had feasted on during the Ice Age died out as a result of global warming and over-hunting.[6] At the same time, edible plants flourished in places that had once been too cold or dry to support them.

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5 http://www.nature.com/nature/journal/v532/n7598/full/nature17176.html

6 http://news.nationalgeographic.com/news/2001/11/1112_overkill_2.html

Based on the behavior of hunter-gatherers in recent times, women did much of the gathering in ancient times and probably used their knowledge of plants to domesticate wheat barley, rice, corn and other cereals. That allowed groups who had once roamed in search of sustenance to settle in one place. The most productive societies those that practiced agriculture by controlling animals and cultivating plants. Agriculture provided food surpluses that allowed people to specialize in other pursuits and devise new tools and technologies.

Some of the earliest advances in agriculture occurred in the Middle East, where sizable towns such as Jericho developed. By 7000 BC Jericho had around 2,000 inhabitants or more than ten times as many people as in a typical band of hunter-gatherers. To protect their community from raiders, the people of Jericho built a wall that became legendary. Within Jericho and other such towns lived many people who specialized in non-agricultural trades, including merchants, metalworkers, and potters. The demand for pots to hold grain and other perishables led to the development of the potter's wheel, which may, in turn, have inspired the first wheeled vehicles. Farmers here and elsewhere used wooden plows pulled by cattle or other draft animals to cultivate their fields and exchanged surplus food for clay pots, copper tools, and other crafted items.

By 5000 BC agriculture was being practiced in large parts of Europe, Asia, and Africa. Few animals were domesticated in the Americas because they had few domestic - able species. (Horses had died out and would not be reintroduced until Europeans reached what they called the New World.) But the domestication of corn and other crops in the Americas led to the growth of villages and complex societies, marked by a high degree of specialization.

2.1.2. Rise and Fall of Civilizations[7,8]

By 3500 BC the stage was set for the emergence of societies so complex and accomplished they rank as civilizations, a word derived from the Latin civic or citizen. All early civilizations had impressive cities or ceremonial centers adorned with fine works of art and architecture. All had strong rulers capable of commanding the services of thousands of people for public projects or military campaigns. Many but not all used writing to keep records, codify laws, and preserve wisdom and lore in the form of literature.

Some of the early historical civilizations that existed were the Harrapan Civilization which lasted from the 3,000 B.C. to 1500 B.C. (Wright 2009), the Egyptian Civilization which nearly lasted for 3000 years (Dodson 2004), the Olmec Civilization which reigned from 1,500 B.C. to 400 B.C. (Malmström 2014) to the recent once like the Roman Empire, in which the western half of its empire lasted from 27 B.C. until 476 A.D. and the eastern half from 330 A.D. until 1453 A.D. (Morris 2010), the Mongolian Empire which lasted from 1206 A.D. to 1368 A.D. (D. Morgan 2007), and many others which saw their fall either due to climate change or invasion from other rulers.[9,10] People in these highly complex societies possessed superior technology, but they were no better or wiser than those in simpler societies.[11] Civilizations embodied the contradictions in human nature.

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7 http://www.historytoday.com/christopher-chippindale/collapse-complex-societies

8 http://www.historyandheadlines.com/top-ten-greatest-civilizations/

9 http://www.mnn.com/earth-matters/climate-weather/blogs/5-ancient-civilizations-were-destroyed-climate-change

10 http://www.gold-eagle.com/article/rise-and-fall-civilizations-0

11 http://www.lanbob.com/lanbob/H-History/HH-02-Societies250KBC-500.htm

They were enormously creative and hugely exploitative, enhancing the lives of some people and enslaving others. Their cities fostered learning, invention, and artistry, but many were destroyed by other so-called civilized people. The glory and brutality of civilization were recognized by philosophers and poets, who knew that anything a ruler raised up could be brought down. "When the laws are kept, how proudly his city stands!" wrote Sophocles. "When the laws are broken, what of his city then?".

Joseph A. Tainter in his book “The Collapse of Complex Societies” defines complex societies in economic and political terms – by the territorial organization, specialized occupations, differentiation in terms of class rather than kinship, a state monopoly of force, of legal jurisdiction, and of authority to direct resources and mobilize people (J. A. Tainter, The Collapse of Complex Societies 1988) . Collapse he defines as a rapid shift to a lower level of complexity. Then he looks at the varied theories of collapse, those that look to external forces of hostile climate change, to internal contradictions of class interest or to the hints of depleted finite resources, or the ones which appeal to mystical or animist analysis, for if a civilization grows and flowers, must it not die in its time also? This, at last, runs back to Vico, whose cyclical theory of history runs from First Barbaric Times to Civil Societies and then to Returned Barbaric Times (Lifshitz 1948).

Tainter, whose viewpoint is from comparative anthropology, has not much patience with tales of morality and redemption. He expects a rational reason to exist for collapse and finds it in economics, generally as a law of declining marginal productivity which will be discussed in the next paragraph. Farming takes the best land first; as farmed area increases so it is forced on to more intractable and less productive land. Mines, which begin with thick and shallow seams, are forced down to thinner and deeper seams. The same is true of social complexity – of civilization – itself. The apparatus of elites, with their ceremonial buildings, luxury goods, warfare and other consumptions are worth their expense so long as there is, overall, a net benefit. So is the expense of conquering neighboring lands, and administering their people. The time comes when extra investment in more complexity and more empire generates no good at all, for the benefits are wholly swallowed up in the costs of supporting the administration, bureaucracy and other parasites that social complexity involves.

Tainter works through three examples to show his general pattern, one from historical sources, two largely from archaeological. The western Roman empire failed because maintaining a far-flung empire in a hostile environment imposed excessive costs on its agricultural basis. The Maya failed because the burdens of competitive warfare and propaganda displays in place of warfare, between the many city-states of the Maya realm could no longer be borne by a weakened population. The Chaco complex, a highly-developed pueblo society of the American southwest of about nine hundred years ago, failed when communities found the costs of contributing to a regional network of redistribution not worth the benefit and withdrew from it. The question becomes less, why do civilizations collapse? and more, why do some civilizations push themselves so far into the regions of greater cost for such small benefit?

2.1.3. IS EARTH FINITE OR INFINITE?

For a very long time researchers have argued the fact of whether we live in a planet with either finite or infinite resource. You might think that the world is finite as the number of atoms, however, big is finite they combine to form a finite number of molecules. The mixture of these molecules might change with time but at the end, it still remains finite. Then you come across economists like John Locke at the end of the 17th century to Adam

Smith in the middle of the 18th century who talked how the planet seemed to be capable of supporting the expansion of the human estate for untold generations to come.[12] They saw the world where there was yet a vast reach of the globe that had to be mapped by humans. Humans relatively were scarce by then and their powers not yet global in scale, not yet amplified by energies of coal and oil. Many other economists still think that our planet is infinite. One of the recent examples is one of the famous economists Milton Mountebank who is said to have revolutionized economic thought, and now he has been recognized for his singular efforts.[13] He demonstrates his idea of infinite planet theory through his book the "*Infinity and Beyond: The Magical Triumph of Economics over Physics"*. Although his book has failed to reach the mainstream audiences, his work has been highly influential among elite political and corporate leaders. Ronald Reagan being a prominent example where once he famously said, "there are no limits to growth and human progress when men and women are free to follow their dreams".[14] That's a close paraphrasing to Mountebank's conclusion to his book.[15]

So, what is the true answer? Is our planet really finite or is it subjective to how we see and interpret growth on this planet? Growth is central to our way of life. Businesses are expected to grow. Every day new businesses are formed and new products are developed. The world population is also growing, so all this adds up to a huge utilization of resources. So, what if at some point, growth in resource utilization collides with the fact

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12 http://www.iep.utm.edu/smith/

13 http://www.steadystate.org/mountebank-nobel/

14 http://www.presidency.ucsb.edu/ws/?pid=38697

15 "Of course, 'Milton Mountebank' is a spoof."

that the world is finite. We have grown up thinking that the world is so large that limits will never be an issue. These are questions that need to be addressed and there exists a reason to develop a method to do so.

2.1.3.1. THE LIMITS TO GROWTH[16]

"The Limits to Growth: a report for the Club of Rome's project on the predicament of mankind" is a 1972 book models economic and population growth with finite resource supplies (Meadows, et al. 1972). The authors were Donella H. Meadows, Dennis L. Meadows, Jorgen Randers and Willian W. Behrens III. The book used a computer model called the World3 model to simulate the consequence of interactions between the earth and human systems. The World3 model was built specifically to invest 5 major trends of global concern - accelerating industrialization, rapid population growth, widespread malnutrition, depletion of nonrenewable resources, and a deteriorating environment. The conclusions from the model were summarized in *"A Report to The Club of Rome (1972)"* by the authors (Limits to Growth: A Report to the Club of Rome, pg. no.1) in 2 points as

(i) "If the present growth trends in world population, industrialization, pollution, food production, and resource depletion continue unchanged, the limits to growth on this planet will be reached sometime within the next one hundred years. The most probable result will be a rather sudden and uncontrollable decline in both population and industrial capacity".

(ii) "It is possible to alter these growth trends and to establish a condition of ecological and economic stability that is sustainable far into the future. The state of global equilibrium could be designed so that the basic material needs of each people on earth are

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16 https://www.bibliotecapleyades.net/sociopolitica/esp_sociopol_clubrome6.htm

satisfied and each people has an equal opportunity to realize his individual human potential".

The World3 model described in the book is a system dynamics model of present trends in a way of looking into the future, especially the very near future, and especially if the quantity being considered is not much influenced by other trends that are occurring elsewhere in the system. Of course, none of the five factors they were examining are independent. Each of the 5 trends constantly interacts with each other like

- Population cannot grow without food
- Food production is increased by growth of capital
- More capital requires more resources
- Discarded resources become pollution
- Pollution interferes with the growth of both population and food

Furthermore, over long periods of time, each of these factors also feedbacks to influence each other. Some projections from the 1972 book using the World3 model can be seen below in Figure 2 along with the observed trends up until 2000.

SIMULATION RESULT FROM 1972, LIMITS TO GROWTH BOOK

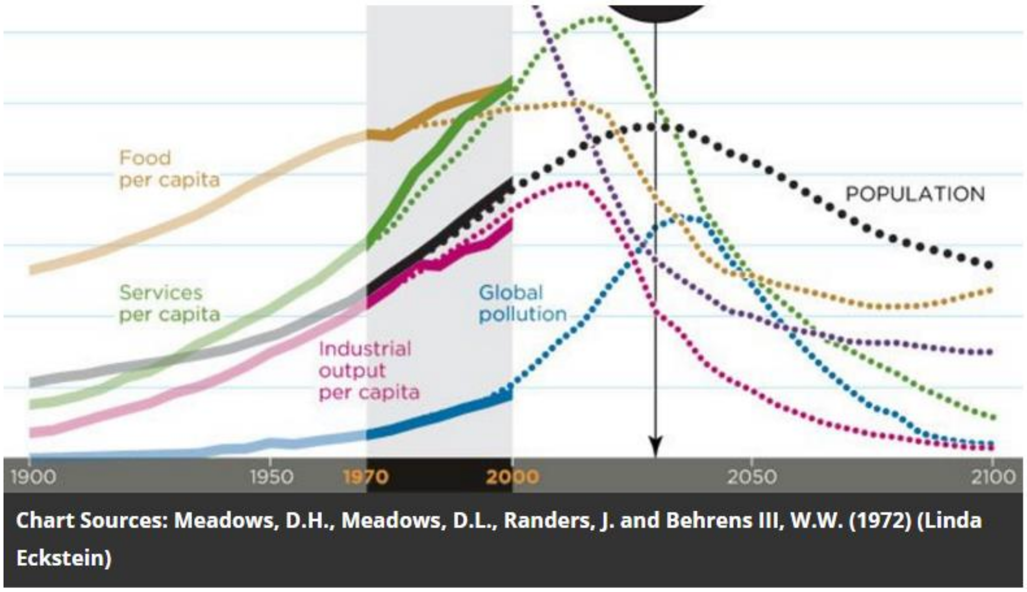

Figure 2. The above figure shows the results from using the World3 model in the 1972 Limits to Growth book along with future predictions on the planet showing it matching a trend.[17]

The Huffington post wrote an article in the year 2016 titled *"What Economists Don't Know about Physics- And why it's Killing us"*.[18] The article summarized the obsession classical economists have towards "growth". They are so fixated are they on growth that recessions are often referred to as periods of "negative growth". The empty promise of perpetual growth is based on folly — and on a fluke of evolutionary history that

17 By Mark Strauss, Smithsonian Magazine; http://www.smithsonianmag.com/science-nature/looking-back-on-the-limits-of-growth-125269840/

18 http://www.huffingtonpost.com/dave-pruett/what-economists-dont-know_b_10742790.html

has allowed humankind to temporarily disregard the laws of physics. Fossil fuels propelled the Industrial Revolution, which in turn gave us mechanization, rapid transportation, the subjugation of nature, and industrial agriculture, all the ingredients for modern, urbanized, high-density societies. Powered first only by coal, then oil came into the picture, and now gas has joined the race, the world's human population more than tripled between 1950 and 2010 (Figure 3).

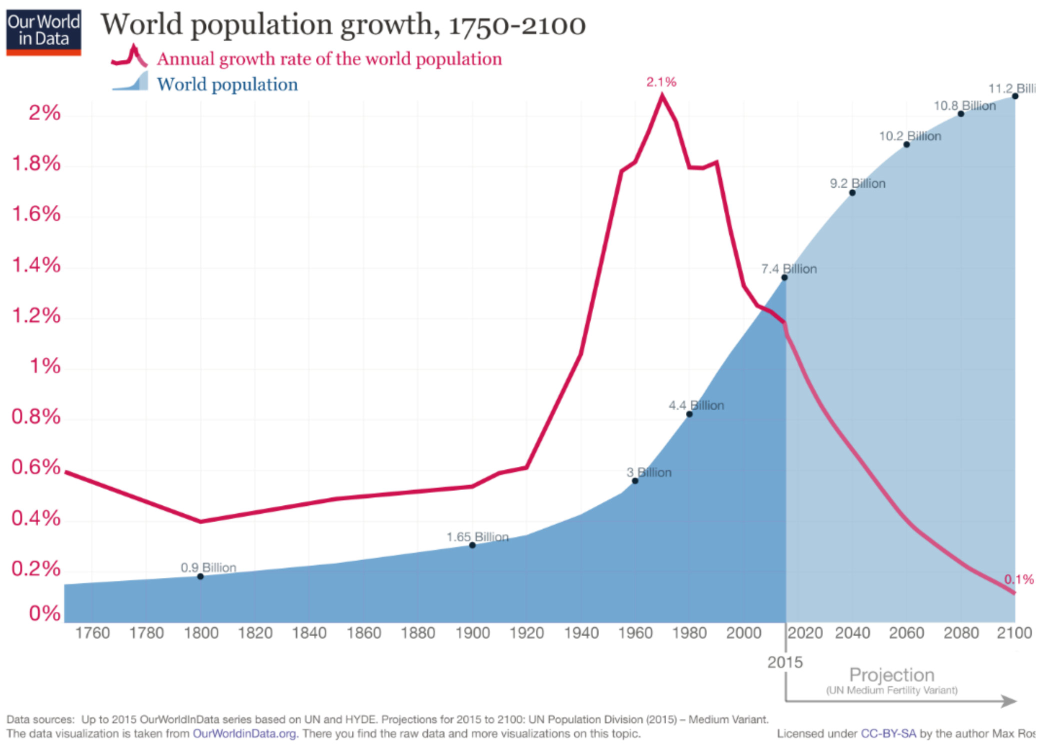

Figure 3. World population 1750-2015 and projections until 2100.[19]

Thermodynamics refer to isolated, closed, and open systems. An isolated system is hermetically sealed: it can exchange neither matter nor energy with its surroundings. At

19 https://ourworldindata.org/world-population-growth/#note-4

the other extreme, an open system can exchange both. With a semi-permeable boundary, a closed system, can exchange energy but not matter. Mainstream economists tend to think of the economy as an abstract, mathematical computerization independent of the physical world. But economic activity necessarily requires natural resources, which are in limited supply as discussed in the opening paragraph. The notion of perpetual economic growth is absurd on face value because it demands an unbounded supply of natural resources and thus an infinitely large earth.[20]

2.1.4. ECONOMIC MODEL

An Economic model (M. S. Morgan 2008) is a theoretical construct representing economic processes by a set of variables and a set of logical and/or quantitative relationships between them. It's a simplified framework designed to illustrate the complex process, often but not always using mathematical techniques. An economic model may have various exogenous variables, and those variables may change to create various responses by economic variables. Methodological uses of models include investigation, theorizing, and fitting theories to the world (M. S. Morgan 2008). These models are used for many purposes like forecasting, economic policy, planning and allocation, logistics, business management etc.

Economic models can be such powerful tools in understanding some economic relationships that it is easy to ignore their limitations (Stanford 1993).

The fundamental issue is circularity: embedding one's assumptions as foundational "input" axioms in a model, then proceeding to "prove" that, indeed, the model's "output" supports the validity of those assumptions. Such a model is consistent with similar models

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20 https://academic.oup.com/nsr/article/3/4/470/2669331/Modeling-sustainability-population-inequality

that have adopted those same assumptions. But is it consistent with reality? As with any scientific theory, empirical validation is needed, if we are to have any confidence in its predictive ability. If those assumptions are, in fact, fundamental aspects of empirical reality, then the model's output will correctly describe reality (if it is properly "tuned", and if it is not missing any crucial assumptions). But if those assumptions are not valid for the particular aspect of reality one attempts to simulate, then it becomes a case of "GIGO" – Garbage In, Garbage Out".

2.1.4.1. SOLOW-SWAN MODEL OF ECONOMIC GROWTH

The Solow–Swan model (Solow 1956) is an exogenous growth model, an economic model of long-run economic growth set within the framework of neoclassical economics. It attempts to explain long-run economic growth by looking at capital accumulation, labor or population growth, and increases in productivity commonly referred to as technological progress. At its core, it is a neoclassical aggregate production function, usually of a Cobb–Douglas type, which enables the model "to make contact with microeconomics" (Acemoglu 2009). The model was developed independently by Robert Solow and Trevor Swan in 1956 and superseded the post-Keynesian Harrod–Domar model (Harrod 1939) and (Domar 1946) . In 1987 Solow was awarded the Nobel Prize in Economics for his work. Today, economists use Solow's sources-of-growth accounting to estimate the separate effects on economic growth of technological change, capital, and labor.

Assumptions in the model:

The key assumption of the neoclassical growth model is that capital is subject to diminishing returns in a closed economy.

(i) Given a fixed stock of labor, the impact on output of the last unit of capital accumulated will always be less than the one before.

(ii) Assuming for simplicity no technological progress or labor force growth, diminishing returns implies that at some point the amount of new capital produced is only just enough to make up for the amount of existing capital lost due to depreciation (Acemoglu, 2009). At this point, because of the assumptions of no technological progress or labor force growth, we can see the economy ceases to grow.

(iii) Assuming non-zero rates of labor growth complicates matters somewhat, but the basic logic still applies (Solow 1956) in the short-run, the rate of growth slows as diminishing returns take effect and the economy converges to a constant "steady-state" rate of growth (that is, no economic growth per-capita).

(iv) Including non-zero technological progress is very similar to the assumption of non-zero workforce growth, in terms of "effective labor": a new steady state is reached with constant output per worker-hour required for a unit of output. However, in this case, per-capita output grows at the rate of technological progress in the "steady-state" (Swan 1956) (that is, the rate of productivity growth).

The Solow-Swan model (Solow 1956) is set in a continuous-time world with no government or international trade. There is only one commodity, output as a whole, whose rate of production is designated $Y(t)$. Thus we can speak unambiguously of the community's real income. Part of each instant's output is consumed and the rest is saved and invested. The fraction of output saved is a constant s, so that the rate of saving is $sY(t)$. The community's stock of capital $K(t)$ takes the form of an accumulation of the composite commodity. Net investment is then just the rate of increase of this capital stock $\frac{dK}{dt}$ or $\dot{K}$, so we have the basic identity at every instant of time:

$$\dot{K} = sY \qquad \text{Eq 1}$$

The output is produced with the help of two factors of production, capital, and labor, whose rate of input is $L(t)$. Technological possibilities are represented by a production function

$$Y = F(K, L) \qquad \text{Eq 2}$$

The output is to be understood as net output after making good the depreciation of capital. About production, all we will say at the moment is that it shows constant returns to scale. Hence the production function is homogeneous of the first degree. This amount to assuming that there is no scarce non-augmentable resource like land. Constant returns to scale seem the natural assumption to make in a theory of growth. The scarce-land case would lead to decreasing returns to scale in capital and labor.

Inserting (Eq 2) in (Eq 1) we get

$$\dot{K} = sF(K, L) \qquad \text{Eq 3}$$

This is one equation in two unknowns. One way to close the system would be to add a demand-for-labor equation: marginal physical productivity of labor equals real wage rate; and a supply-of-labor equation. The latter could take the general form of making labor supply a function of the real wage equal to a conventional subsistence level. In any case, there would be three equations in the three unknowns $K, L, real\ wage$.

Instead, if we proceed more in the spirit of the Harrod's model (Harrod 1939). As a result of exogenous population growth, the labor force increases at a constant relative rate n. In the absence of technological change n is Harrod's natural rate of growth. Thus

$$L(t) = L_o e^{nt} \qquad \text{Eq 4}$$

Alternatively, (Eq 4) can also be looked as a supply curve of labor. It says that the exponentially growing labor force is offered for employment completely in-elastically. The labor supply curve is a vertical line which shifts to the right in time as the labor force grows according to (Eq 4). Then the real wage adjusts so that all available labor is employed, and the marginal productivity equation according to (Eq 5) and (Eq 6) determines the wage rate which will actually rule.

$$\frac{\partial F}{\partial L} = \frac{w}{p} \qquad \text{Eq 5}$$

$$\frac{\partial F}{\partial K} = \frac{q}{p} \qquad \text{Eq 6}$$

In (Eq 3) L stands for total employment whereas in (Eq 4) L stands for the available supply of labor. By identifying the two we are assuming that full employment is perpetually maintained. When we insert (Eq 4) in (Eq 3) to get

$$\dot{K} = sF(K, L_o e^{nt}) \qquad \text{Eq 7}$$

We have this basic equation that determines the time evolution of capital accumulation that must be followed if all available labor is to be employed. In summary, (Eq 5) is a differential equation in the single variable $K(t)$. Its solution gives the only time profile of the community's capital stock that can fully employ the available labor. Once we know the time evolution of capital stock and that of the labor force, we can compute from the production function the corresponding time path of real output.

2.1.4.2. COBB-DOUGLAS MODEL OF PRODUCTION

In economics, the Cobb–Douglas (Cobb and Douglas 1928) and (Douglas 1976) production function is a particular functional form of the production function, widely used to represent the technological relationship between the amounts of two or more inputs, particularly physical capital and labor, and the amount of output that can be produced by those inputs. Sometimes the term has a more restricted meaning, requiring that the function display constant returns to scale (in which case $\beta = 1 - \alpha$ in the formula below). In its most standard form for production of a single good with 2 factors, the function is

$$Y = AL^{\beta}K^{\alpha} \qquad \text{Eq 8}$$

Where,

$$Y = Total\ poduction\ (\ the\ real\ value\ of\ all\ goods\ produced\ in\ a\ year)$$

$$L = Labor\ Input\ (\ the\ total\ number\ of\ persons - hours\ worked\ in\ a\ year)$$

$$K = Capital\ input\ \begin{pmatrix} the\ real\ value\ of\ all\ machinery, \\ equipment, and\ buildings \end{pmatrix}$$

$$A = total\ factor\ productivity$$

$\alpha\ and\ \beta\ are\ the\ output\ elasticities\ of\ capital\ and\ labor, respectively.$

If

$$\alpha + \beta = 1, \qquad \text{Eq 9}$$

The production function has constant returns to scale, meaning that doubling the usage of capital K and Labor L will also double output Y. If

$$\alpha + \beta < 1, \qquad \text{Eq 10}$$

Returns to scale are decreasing, and if

$$\alpha + \beta > 1, \qquad \text{Eq 11}$$

Returns to scale are increasing. Assuming perfect competition[21] and $\alpha + \beta = 1, \alpha\ and\ \beta$ can be shown to be capital's and labor's share of output.

2.1.4.3. LEONTIEF PRODUCTION FUNCTION

The Leontief production function or fixed proportions production function is a production function that implies the factors of production will be used in fixed (technologically pre-determined) proportions, as there is no substitutability between factors. It was named after Wassily Leontief and represents a limiting case of the constant elasticity of substitution production function (Arrow, et al. 1961), (Jorgensen 2000) and (Klump, William and McAdam 2007)

For the simple case of a good that is produced with two inputs, the function is of the form

$$q = Min(\frac{z_1}{a}, \frac{z_2}{b}) \qquad \text{Eq 12}$$

where q is the quantity of output produced, z1 and z2 are the utilized quantities of input 1 and input 2 respectively, and a and b are technologically determined constants.

2.2. GAPS IN LITERATURE

All economic models, no matter how complicated, are subjective approximations of reality designed to explain observed phenomena. It follows that the model's predictions must be tempered by the randomness of the underlying data it seeks to explain and by the validity of the theories used to derive its equations. A good example would be highlighting the failure of existing models to predict or untangle the reasons for the global financial crisis that began in 2008. Insufficient attention to the links between overall demand, wealth,

21 http://economictimes.indiatimes.com/definition/perfect-competition

and in particular excessive financial risk taking has been blamed.[22] No economic model can be a perfect description of reality. Economic models can also be classified in terms of the regularities they are designed to explain or the questions they seek to answer. For example, some models explain the economy's ups and down around an evolving long-run path, focusing on the demand for goods and services without being too exact about the sources of growth in the long run. Other models are designed to focus on structural issues, such as the impact of trade reforms on long-term production levels, ignoring short term oscillations.

Keynesian economics tells us that how in the short run, and especially during recessions, economic output is strongly influenced by aggregate demand.[23] In the Keynesian view, aggregate demand does not necessarily equal the productive capacity of the economy; instead, it is influenced by a host of factors and sometimes behaves erratically, affecting production, employment, and inflation. The theories forming the basis of Keynesian economics were first presented by the British economist John Maynard Keynes during the Great Depression in his 1936 book, *"The General Theory of Employment, Interest, and Money"* (Keynes 1936). Keynesian economists often argue that private sector decisions sometimes lead to inefficient macroeconomic outcomes which require active policy responses by the public sector, in particular, monetary policy actions by the central bank and fiscal policy actions by the government, in order to stabilize output over the business cycle. Keynesian economics advocates a mixed economy – predominantly private sector, but with a role for government intervention during

22 http://www.imf.org/external/pubs/ft/fandd/basics/models.htm

23 http://www.imf.org/external/pubs/ft/fandd/2014/09/basics.htm

recessions. Keynesian economics served as the standard economic model in the developed nations during the latter part of the Great Depression, World War II, and the post-war economic expansion (1945–1973), though it lost some influence following the oil shock and resulting stagflation of the 1970's. The advent of the financial crisis of 2007–08 caused a resurgence in Keynesian thought, which continues as new Keynesian economics (Dixon 1999).

Steve Keen is a professor of Economics at Kingston University London who writes a blog called Steve Keen's Debtwatch where he wrote an article titled "Neoclassical Economics: mad, bad and dangerous to know", which was published in 2009 (Keen, Neoclassical Economics: mad, bad, and dangerous to know 2009). In this article, he highlights how the most important thing that global financial crisis has done for economic theory is to show that neoclassical economics is not merely wrong, but dangerous. He justifies this by arguing how neoclassical economics contributed directly to the 2007-08 crisis by promoting a faith in the innate stability of a market economy, in a manner which in fact increased the tendency to instability of the financial system, with its false belief that all instability in the system can be traced to interventions in the market, rather than the market itself, it championed the deregulation of finance and a dramatic increase in income inequality. Its equilibrium vision of the functioning of finance markets led to the development of the very financial products that are now threatening the continued existence of capitalism itself.

Simultaneously it distracted economists from the obvious signs of an impending crisis—the asset market bubbles, and above all the rising private debt that was financing them. Paradoxically, as capitalism's "perfect storm" approached, neoclassical macroeconomists were absorbed in smug self-congratulation over their apparent success in taming inflation and the trade cycle, in what they termed “The Great Moderation”.

3. Chapter 3: Research Approach

3.1. Research Approach

All models are wrong, and some are useful. The neoclassical economic framework has proved useful for many purposes. But today's challenges are those of new constraints pushing the boundaries of economic thinking. Central banking interest rates are lower (now negative in some cases) than any time in the history of central banking, and debt levels are at all-time highs. Also, global food and energy are no longer getting cheaper (King 2015b) after continuous decreases since the industrial revolution. Narratives and models exist for describing past agrarian civilizations and their relation to resource access (J. A. Tainter, The Collapse of Complex Societies 1988), (J. A. Tainter, Energy, Complexity, and Sustainability: A Historical Perspective 2011) and (Tainter, et al. 2003) as well dynamics of population and social structure (Turchin, Currie, et al. 2013) and (Turchin and Nefedov, Secular Cycles 2009) These narratives often describe how the quality and quantity of natural resources relate to the cyclical growth, structure, inequality, and/or complexity of society and/or the economy overall.

However, it is unclear how these narratives translate both to today's modern society enabled by fossil fuels and to a low-carbon energy economy transition. Past agricultural societies rose and fell powered essentially by renewable energy flows (e.g., sunlight). These societies were also very unequal with a small number of elite collecting the vast majority of the income while the vast majority of commoners worked in the energy and food (e.g., agricultural) sectors. Fossil-fueled machinery, fertilizers, and irrigation enabled energy and food costs to continually decrease since the start of industrialization. Thus, today developed economies, primarily powered by fossil fuel stocks, have a very small proportion of workers in energy and food sectors with an equal income and resource access as compared to the preindustrial world (Mitchell 2013). If the food and energy costs have reached their cheapest points then many current macroeconomic modeling approaches, often calibrated only to relatively recent fossil-fueled history, might be insufficient for understanding current and future economic growth with regard to constraints within food, water, and energy sectors; climate change; and debt (Pindyck 2013), (Stern 2013) and (Keen, A Monetary Minsky Model of the Great Recession 2013a). As we attempt to transition to a low-carbon energy supply, largely based on renewable energy flows, it is paramount to have internally consistent macro-scale models that track flows and interdependencies among money, debt, employment and biophysical quantities (e.g., natural resources and population).

What if you realized that the fundamental economic framework of models that are meant to guide a low-carbon energy transition prevents them from actually answering the question they are supposed to answer? Instead of assuming a series of energy investments, and then estimating the economic impacts of those choices, they actually do the exact opposite. They assume economic growth and then make a series of investments to meet emissions targets without actually factoring in how the energy systems themselves feedback to economic growth or other social goals.

Monetary models of finance and debt assume that energy resources and technology are not constraints on the economy. Energy transition scenario models assume that economic growth, finance, and debt will not be constraints on the energy transition. These assumptions must be eliminated, and the modeling concepts must be integrated if we are to properly plan for and understand the dynamics of a low-carbon and/or renewable energy transition.

Over the past 200 years, human civilization has transformed from one dependent upon renewable energy flows (e.g., sunlight) to one dependent upon fossil energy stocks (e.g., oil, gas, and coal). Climate change and resource depletion are driving society to understand how to again live off of low-carbon renewable flows of primary energy. Except for this time, we are much smarter, and we have increased technological know-how.

As we attempt to transition to a low-carbon energy supply, largely again based again on renewable energy flows, it is paramount to have internally consistent macro-scale models that track flows and interdependencies among money, debt, employment and biophysical quantities (e.g., natural resources and population). This research objective is to develop a framework to describe our contemporary and future macroeconomic situation that is consistent with both biophysical and economic principles. Unfortunately, this fundamental integration does not underpin our current thinking. This improved framework can contribute to more robust policy-making ability under both current and future changing circumstances.

The objective here is to develop a consistent biophysical and economic framework to describe the industrial transition to our contemporary macroeconomic state. This research seeks to integrate macro-scale system dynamics models of money, debt, and employment (specifically the Goodwin and Minsky models of (Keen, Finance and Economic Breakdown: Modeling Minsky's "Financial Instability Hypothesis" 1995)) with system dynamics models of biophysical quantities (specifically population and natural resources such as in (Meadows, et al. 1972) and (Motesharrei, Rivas and Kalnay 2014). In other words, there are models of each separately, but they have not been combined to fundamentally link the biophysical world to monetary frameworks. Figure 4 outlines the approach as a critical extension of existing literature.

This type of modeling can answer important questions for a low-carbon transition (two examples):

1. How does the rate of transition feedback to the growth of population, economic output, and debt?

The faster we transition, the more capital, labor, and natural resources will be mobilized to become part of the "energy" sectors. The larger this mobilization, the higher the cost of energy will become as there is an increasing number of "energy" sector workers dependent upon selling energy to a decreasing set of "non-energy" sector consumers. Increasing labor and capital shares for energy is the exact opposite trend of industrialization as we know it, and there is a critical need to understand the associated feedbacks. For example, the most recent oil and gas boom and bust cycle could possibly be explained by too many resources being allocated to the energy sector over too short of a time for the rest of the economy to adjust. It is important we understand this growth feedback between the size (labor, capital, energy) of energy and food sectors and economic growth.

2. How do the capital structure (e.g., fixed costs versus variable costs) of fossil and renewable energy systems relate to and affect economic outcomes?

Renewable and low-carbon energy systems (e.g., PV, the wind, nuclear, electrochemical storage) are characterized by a much higher fraction of fixed (capital) costs as compared to fossil energy systems (e.g., coal, natural gas, and oil). Higher fixed costs systems are more favorable in certain (e.g., predictable) and lower growth (with low discount rate) environments whereas lower fixed cost systems are more favorable in uncertain and high growth situations (Chen 2016). The reason is that in high growth situations you do not want to be "stuck" with old capital in which you are still waiting for returns to reinvest. Low economic growth, associated with low discount rates, also make high fixed cost and longer-life assets, like renewable systems, more favorable. Thus, we should expect low growth ("secular stagnation") to be associated with low-interest rates and high renewable energy installations, just as has happened over the last several years.

RESEARCH APPROACH

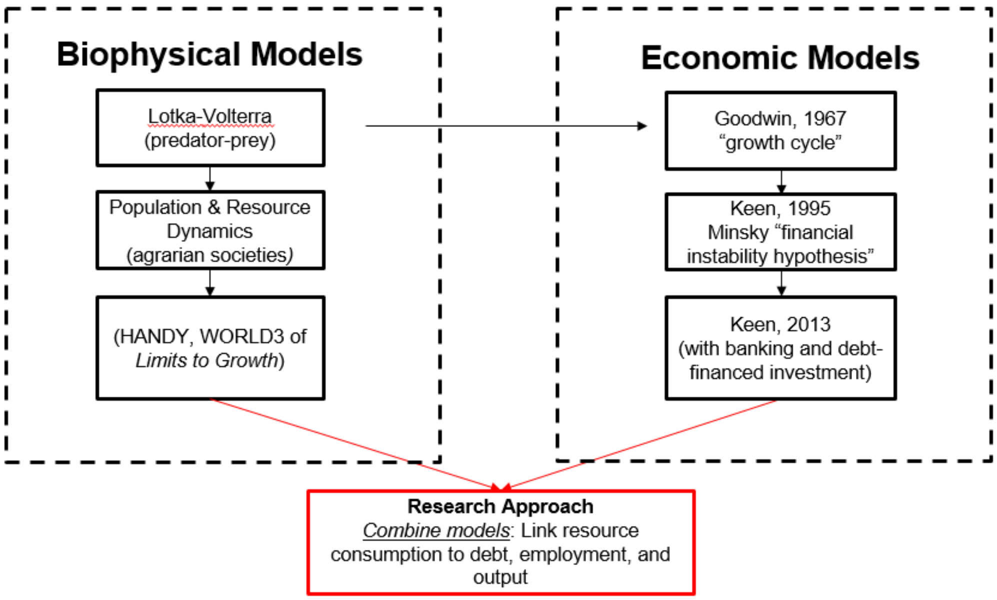

Figure 4. The research approach will focus to make a critical link between biophysical modeling concepts and those of economic models that specifically include the link of debt-based finance to employment and economic growth.

3.2. Background and Context

3.2.1. Lotka-Volterra-Predator-Prey Model

The Predator–Prey model, was derived independently by two mathematicians Alfred Lotka and Vito Volterra, in the early 20th century (Lotka 1925). This model describes the dynamics of competition between two species, say, wolves and rabbits. The governing system of equations is

$$\frac{dx}{dt} = (ay)x - bx \qquad \text{Eq 13}$$

$$\frac{dy}{dt} = cy - (dx)y \qquad \text{Eq 14}$$

In the above system, x represents the predator (wolf) population; y represents the prey (rabbit) population; a determines the predator's birth rate, i.e., the faster growth of wolf population due to availability of rabbits; b is the predator's death rate; c is the prey's birth rate; d determines the predation rate, i.e., the rate at which rabbits are hunted by wolves. Rather than reaching a stable equilibrium, the predator and prey populations show periodic, out-of-phase variations about the equilibrium values

$$x_e = \frac{c}{d} \qquad \text{Eq 15}$$

$$y_e = \frac{b}{a} \qquad \text{Eq 16}$$

Note the consistency of the units on the left and right-hand sides of Eq 13 through Eq 16. A typical solution of the predator–prey system can be seen in Figure 5.

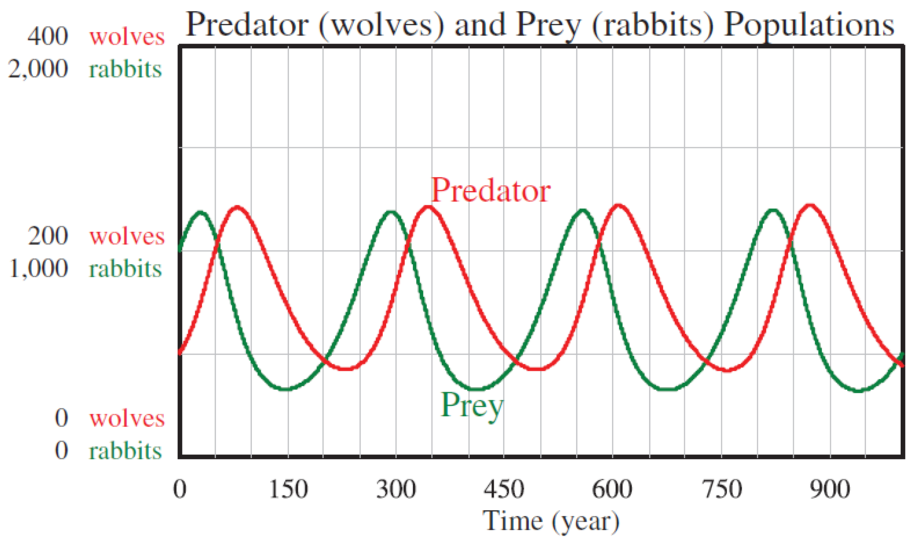

Figure 5. A typical solution of the predator-prey system (Motesharrei, Rivas and Kalnay 2014, 3) obtained by running the system with a certain set of initial conditions for the number of wolves and rabbits at the start of the simulation.

3.2.2. MULTIPLIER-ACCELERATOR MODEL

The Multiplier-Accelerator (Samuelson 1939) Interaction Theory came into existence when the theorist of the Keynesian tradition stresses on multiplier process in economic fluctuations while J.K. Clark emphasized on the role of acceleration in the business fluctuations.

But however, Paul Samuelson, the post-Keynesian business cycle theorists asserted that neither the multiplier theory nor the principle of acceleration alone is adequate to analyze the business cycle fluctuations. And hence, proposed the Multiplier-Accelerator Model, also called as Hanson-Samuelson Model.

The multiplier-Accelerator model is based on the Keynesian multiplier, a consequence of the assumption that the level of economic activity decides the consumption intentions and the accelerator theory of investment which is based on the assumption that the investment intentions depend on the pace with which the economic activities grow.

The Samuelson's model is the first step towards integrating the theory of multiplier and principle of acceleration. This theory shows how well these two tools are integrated to generate income, so as to have an increased consumption and investment demands more than expected and how these reflect the changes in the business cycle. To understand Samuelson's model one needs to know the difference between the Autonomous and Derived Investments.

Autonomous investment is the investment undertaken due to the external factors such as new inventions in technology, production process, production methods, etc. While, the Derived Investment is the investment, particularly in capital equipment, is undertaken to meet the increase in consumer demand necessitating new investment.

With an increase in the autonomous investment, the income of people rises and the process of multiplier begins. With increased incomes, the demand for the consumer goods also increases depending on the marginal propensity to consume. And if the firm has no excess production capacity, then its existing capital will stand inadequate to meet the increased demand. Therefore, the firm will undertake new investment to meet the growing demand. Thus, an increase in consumption creates a demand for investment, and this is called as Derived Investment. This marks the beginning of the acceleration process.

When the derived investment takes place, the income rises, in the same manner, it does when the autonomous investment took place. With an increased income, the demand for the consumer goods also increases. This is how, the multiplier process and principle of acceleration interact with each other, such that income grows at a faster rate than expected.

In short, the exogenous factors (external origin) lead to autonomous investment, which results in the multiplier effect. This multiplier effect creates the derived investment, which results in the acceleration of investment. Samuelson made the following assumptions in the analysis of this interaction process:

1. There is no excess production capacity.
2. At least one-year lag in the consumption.
3. At least one-year lag in the increase in demand for consumption and investment.
4. No government intervention, and no foreign trade.

Samuelson's model of business fluctuations is presented below:

Given the assumption (4) as above, the economy is said to be in equilibrium when,

$$Y_t = C_t + I_t \qquad \text{Eq 17}$$

Where, Y_t= national income, C_t = total consumption expenditure, I_t = total investment expenditure, all in a period 't'.

Given the assumption (2) as above, the consumption function can be expressed as

$$C_t = aY_{t-1} \qquad \text{Eq 18}$$

Where, Y_{t-1}= income in period t-1, and a = marginal propensity to consume

Investment is a function of consumption with one year lag, and is expressed as:

$$I_t = b(C_t - C_{t-1}) \qquad \text{Eq 19}$$

Where, b = capital/output ratio. Here parameter 'b' determines the accelerator.

By substituting equation (Eq 18) for C_t and equation (Eq 19) for I_t, the equilibrium equation can be written as:

$$Y_t = aY_{t-1} + b(aY_{t-1} - aY_{t-2}) \qquad \text{Eq 20}$$

Further, simplifying the equation:

$$Y_t = a(1+b)Y_{t-1} - abY_{t-2} \qquad \text{Eq 21}$$

Samuelson's model suffers from the following criticisms:

1. The critics feel that it is far too simple a model to explain what all happens during the economic fluctuations. They are of an opinion that the model has been developed on highly simplifying assumptions.
2. Samuelson stresses on the role of multiplier and accelerator and the interaction between them as a fundamental cause of business fluctuations. Thus, like other theories, it has also ignored the other important factors that play a crucial role in a cyclical process, such as producer's expectations, change in the psychology of businessmen, change in consumer's tastes and preferences and the exogenous factors.
3. One of the major criticism of this model is that it is assumed that the capital/output ratio remains constant while there are chances of change in this ratio during expansion and depression.
4. Finally, the cyclical patterns suggested in this model do not confirm the real-world experience.

In spite of these bottlenecks, a Samuelson's model is acclaimed as a sound attempt to integration between the Keynesian multiplier theory and Clarke's acceleration principle, that fairly explains the causes of fluctuations in the business cycles.[24]

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24 Luca Fiorito "John Maurice Clark's Contribution to the Genesis of the Multiplier Analysis," University of Siena Dept. of Econ. Working Paper No. 322

3.2.3. THE GOODWIN MODEL[25,26]

In 1967, Richard Goodwin developed an elegant model meant to describe the evolution of distributional conflict in growing, advanced capitalist economies. The Goodwin model is important because it tells a story about the dynamics of the growth and distribution process at the heart of the Foley and Michl approach (Foley and Michl 1999).

The key variables in the Goodwin model are the share of labor in national income, and the employment rate. In the conventional wage share model of Foley and Michl, the labor share is always constant at its conventional level. While this is a reasonable approximation over the very long-run, the labor share fluctuates in actual economies (Goodwin 1967). Similarly, in the conventional wage share model, the amount of labor hired each period is given by $\rho K/x$ where ρ is the output/capital ratio, x is labor productivity, and K is capital.

The economic intuition is the following: Suppose the economy is expanding, and employment increases. Higher labor demand generates wage inflation which, as long as real wages increase more than labor productivity, increases the wage share in output. If consistently with the Classical view, workers do not save, the resulting decrease in the profit share will act in reducing future investment and output. But then the economy is down, and lower demand for labor will then correspond to lower output, leading the way to lower wage inflation or even wage deflation. The labor share will decrease. But a higher profit share will produce a surge in investment, which will generate higher employment,

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25 http://www.danieletavani.com/wp-content/uploads/2014/05/ECON705_Goodwin.pdf

26 http://orion.math.iastate.edu/driessel/14Models/1967growth.pdf

thus improving workers bargaining power and consequently wages. At this point, the wage share has increased, and the cycle can repeat itself.

Goodwin realized that this type of dynamics is also found in simple biological systems, such as the Lotka-Volterra Predator-Prey model described in Section 3.2.1. Suppose that in a certain territory there is a predator species and a prey species. If predators are too little in number, Prey’s will proliferate, thus pushing against the resource constraint in the territory. Prey proliferation, however, makes predators' life easier: they will be better fed and reproduce at a higher rate. But when the number of predators is too high, finding preys to eat becomes harder, and mortality among predators will increase. Prey’s will have more opportunities to reproduce, and the cycle can repeat itself.

Assumptions in the model

1. steady technical progress (disembodied);[27]
2. steady growth in the labor force;
3. only 2 factors of production, labor and “capital” (plant and equipment), both homogeneous and non-specific;
4. all quantities real and net;
5. all wages consumed;
6. all profits saved and invested;

These assumptions are of a more empirical, and disputable, sort:

7. a constant capita-output ratio;
8. a real wage rate which rises in the neighborhood of full employment

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[27] Disembodied Technical Progress: Improved technology which allows increase in the output produced from given inputs without investing in new equipment.

3.2.3.1. MATHEMATICS OF GOODWIN MODEL

Goodwin's model can be expressed as 2 differential equations in the rate of employment and the wages share of output. (Keen, A Monetary Minsky Model of the Great Moderation and the Great Recession 2013) outlines it in terms of absolute values, since this is the form in which the final monetary Minsky model in this research is also expressed as also in Keen's 2013 paper. The economy is populated by workers and capitalists. Workers supply labor services to firms owned by capitalists, who do not save and consume all of their income. Capitalists own capital assets, consume and save. The aggregate output is denoted by Y, capital by K and labor by L. Labor is homogeneous, hence a single wage rate w can be used. Production of output occurs with the following fixed proportions for technology:

$$Y = \min\{\frac{K}{v}, aL\} \qquad \text{Eq 22}$$

Implying that the real unit labor cost which is the labor share can be represented as $\frac{w}{a}$. Labor productivity grows at the exogeneous rate $\alpha>0$, while the capital to output ratio remains constant over time. The model also assumes a exogenous growth of the labor force, at a rate $\beta>0$. Since full employment might or might not occur, we denote the employment rate by $\lambda = L/N$, where N is the total population/ labor force in the model

Keen outlines Goodwin's equation in terms of absolute values since this is the form in which the final monetary Minsky model in his paper (Keen, A Monetary Minsky Model of the Great Moderation and the Great Recession 2013) is expressed. The basic deterministic casual cycle of the model is:

1. The level of capital stock K determines the level of output per annum Y via the accelerator v:

$$Y = \frac{K}{v} \qquad \text{Eq 23}$$

2. The level of output determines the level of employment L via labor productivity a:

$$L = \frac{Y}{a} \qquad \text{Eq 24}$$

3. The level of employment determines the employment rate λ (the ratio of L to population N):

$$\lambda = \frac{L}{N} \qquad \text{Eq 25}$$

4. The employment rate determines the rate of change of real wages w via a Phillips curve:

$$\frac{1}{w}\frac{dw}{dt} = -c + d\lambda \qquad \text{Eq 26}$$

5. Profit determines investment I (in the Goodwin model, all profits are invested):

$$I = \prod \qquad \text{Eq 27}$$

6. Investment minus depreciation γ determines the rate of change of capital stock K, closing the model:

$$\frac{dK}{dt} = I - \gamma.K \qquad \text{Eq 28}$$

With population growth of β percent per annum, labor productivity growth of α percent per annum, and a linear Phillips curve relation of the form $(-c + d\lambda)$ (where c and d are constants) (Phillips 1958), the model consists of the following 4 differential equations in the level of employment, the real wage, labor productivity and population growth. A typical simulation of the Goodwin model can be seen in Figure 6.

1. $\frac{dL}{dt} = L(\frac{\left(1-\frac{w}{a}\right)}{v} - \gamma - \alpha)$ Eq 29
2. $\frac{dw}{dt} = (-c + d\lambda)w$ Eq 30
3. $\frac{da}{dt} = \alpha a$ Eq 31
4. $\frac{dN}{dt} = \beta N$ Eq 32

Eq 29 can also be modeled in other forms where it can be represented as Eq 33 & Eq 34 below in accord with the condition in Eq 22 when Y $= \frac{K}{v} = al$ under full capital utilization (Keen, Finance and Economic Breakdown: Modeling Minsky's "Financial Instability Hypothesis" 1995) and therefore,

$$\frac{dY}{dt} = Y.(\frac{\left(1-\frac{w}{a}\right)}{v} - \gamma) \quad \text{Eq 33}$$

and,

$$\frac{dK}{dt} = K.(\frac{\left(1-\frac{w}{a}\right)}{v} - \gamma) \quad \text{Eq 34}$$

SIMULATION RESULTS FROM THE GOODWIN MODEL

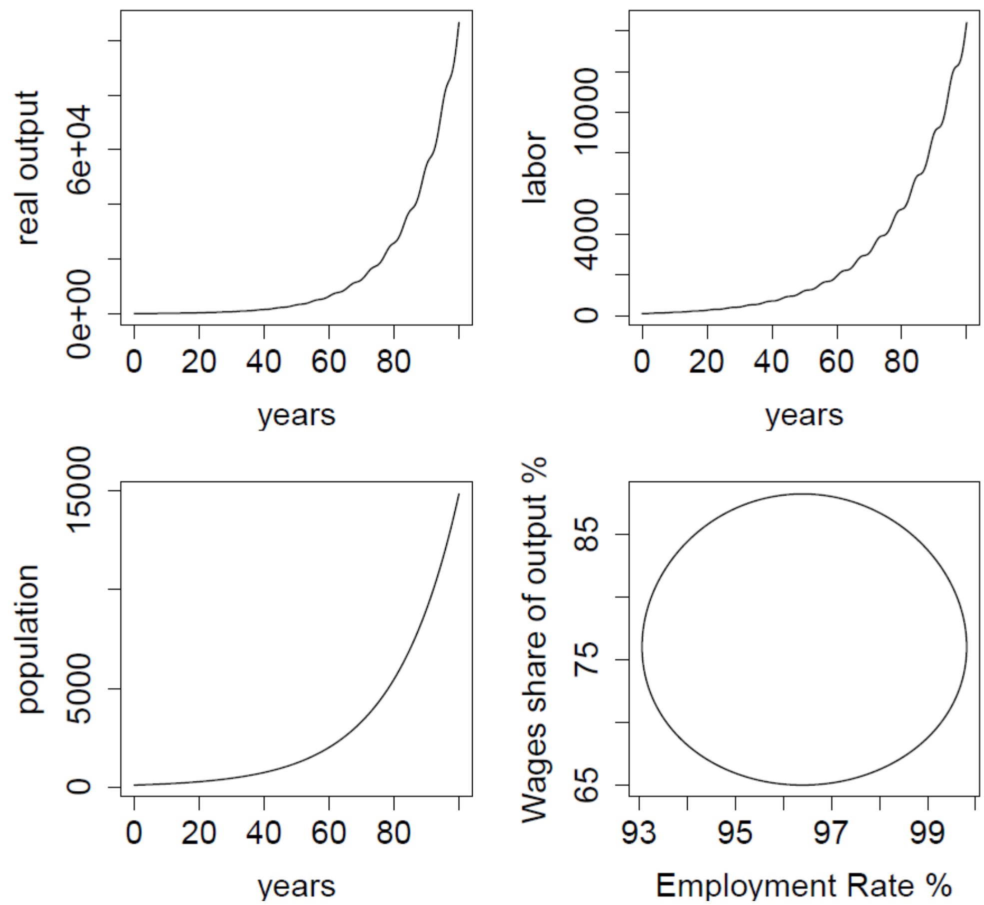

Figure 6. Goodwin model with the Linear form for the Phillips curve (-c + d λ) where c = 4.8 and d =5 are constants and λ is the employment rate with an initial value of 0.97. The initial value for the real wage rate w is equal to 0.88 and that for the labor productivity a is equal to 1.

3.2.4. Human and Nature Dynamics (“HANDY”): Modeling inequality and use of resources in the collapse or sustainability of societies

3.2.4.1. The Model

The HANDY model (Motesharrei, Rivas and Kalnay 2014) is inspired from the predator – prey model where instead of modeling two different species we have

1. The human population which comprises of the elites and commoners in the model and

2. The available nature which is the total nature available in the system which is consumed by the population in the model.

The HANDY paper defines the population to address the effect of inequality towards the sustenance of a particular society. There are 4 basic equations which describe the entire idea:

1. State Variables
    a. Population of commoners (x_c)
    b. Population of elites (x_e)
    c. Nature (y)
    d. Wealth (w)
2. State equations
    a. Rate of change of commoner’s population

$$\frac{d}{dt}x_c = \beta_c x_c - \alpha_c x_c \qquad \text{Eq 35}$$

    b. Rate of change of elite’s population

$$\frac{d}{dt}x_e = \beta_e x_e - \alpha_e x_e \qquad \text{Eq 36}$$

c. Rate of change of nature available for extraction

$$\frac{d}{dt}y = \gamma\, y\,(\lambda\, - y\,) - \delta x_c y \qquad \text{Eq 37}$$

d. Rate of change of wealth accumulated due to extraction of nature

$$\frac{d}{dt}w_h = \delta\, x_c y - \, C_c - \, C_e \qquad \text{Eq 38}$$

System Interactions in the "HANDY" model

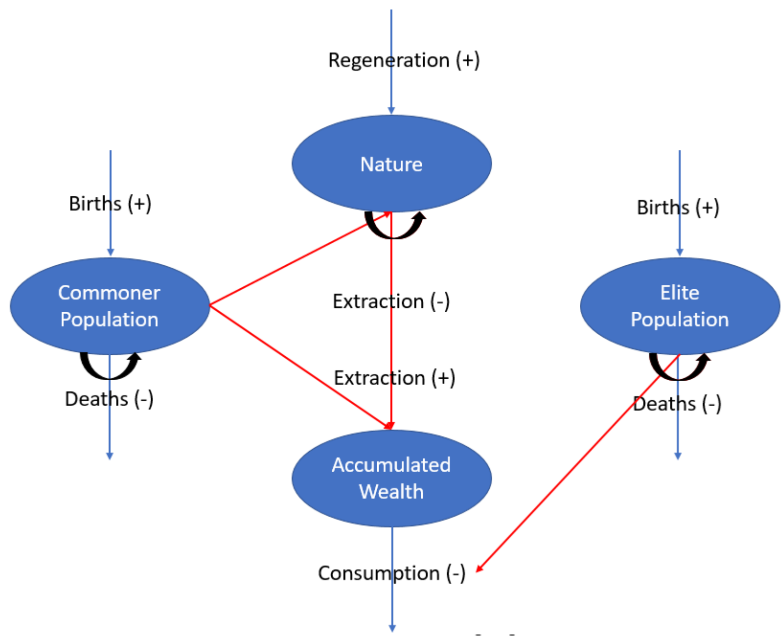

Figure 7. The above diagram shows flows and stocks in the "HANDY" model.

Assumptions in the model:

1. The population grows through a birth rate β and decrease through a death rate α. The birth rate β is considered the same for both the elites and the commoners in the model.

2. The commoners are the only one's working in the system towards extraction of nature which in return gets stored as accumulated wealth which is the final thing consumed by both the commoners and elites.
3. Both available nature and accumulated wealth have similar units called as eco$
4. Consumption by both commoners and elites are different with elites consuming higher than the commoners by a factor κ. Their consumption is given by

a. $$C_c = \min\left(1, \frac{w}{w_{th}}\right) s x_c \qquad \text{Eq 39}$$

b. $$C_e = \min\left(1, \frac{w}{w_{th}}\right) s \kappa x_e \qquad \text{Eq 40}$$

$$w_{th} = \rho(x_c + \kappa x_e) \qquad \text{Eq 41}$$

Eq 41 above is the wealth threshold value of wealth below which famine starts and ρ is the minimum required consumption per capita. The death rates of both elites and commoners are a function of their consumptions and expressed in terms of $\frac{w}{w_{th}}$ (Figure 8) and the graphical representation of the death rates can be seen below in Figure 9.

c. $$\alpha_C = \alpha_m + \max(0, 1 - \frac{C_C}{s x_C})(\alpha_M - \alpha_m) \qquad \text{Eq 42}$$

d. $$\alpha_E = \alpha_m + \max(0, 1 - \frac{C_E}{s \kappa x_C})(\alpha_M - \alpha_m) \qquad \text{Eq 43}$$

Where,

$C_c = Consumption\ by\ Commoners$, $C_e = Consumption\ by\ Elites$

$\alpha_C = Commoner\ Death\ Rate$, $\alpha_e = Elite\ Death\ Rate$

$\alpha_m = Normal\ (minimum)\ Death\ Rate, \quad \alpha_M = Famine\ (maximum)\ Death\ Rate$

with the condition that

$$\alpha_m < \beta_e < \beta_C < \alpha_M \qquad \text{Eq 44}$$

implying that the birth rates cannot be higher than the maximum death rate and less than the minimum death rate in the model.

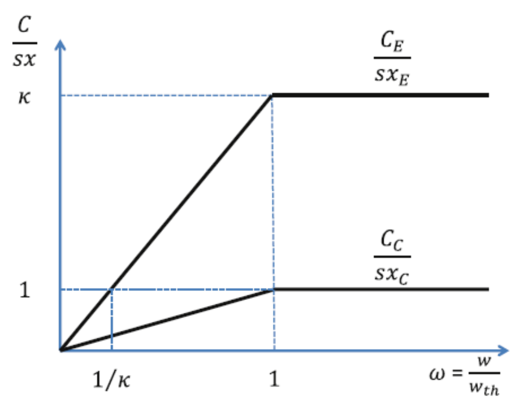

Figure 8. Consumption rates in HANDY.

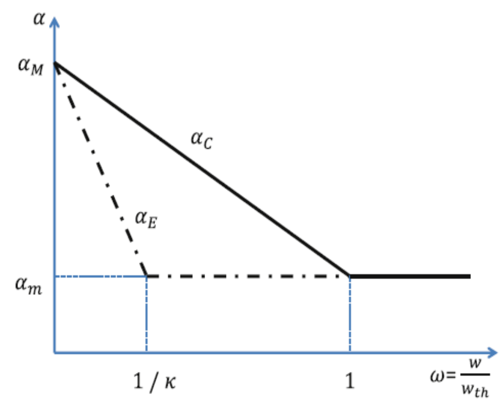

Figure 9. Death rates in HANDY.

3.2.4.2. DIFFERENT SCENARIOS IN THE MODEL

Motesharrei et al. 2014 uses the HANDY model to explore different scenarios by varying the factors (i) and (ii) below in order to explore different scenarios. They do this in order to explore two different types of possible collapses in the society either due to excess/rapid extraction of nature or high inequality in the society

(i) Rate of depletion of nature δ
(ii) Inequality Factor κ

One such scenario is of an Egalitarian society: An Egalitarian society in the paper is defined as a society with no elites in the model and therefore $x_e = 0$. Results from simulating such scenarios can be seen in Figure 10 and Figure 11 with two different rates for the depletion of nature δ. At lower values of δ the society reaches a steady state over a long period of time. Whereas, at higher values a, collapse is seen in the society when the population overshoots the carrying capacity in the model. The carrying capacity in the model is defined as the maximum population the society can sustain after which it starts falling due to lack of availability of nature. Another scenario that has been discussed in the paper is of an Equitable Society Figure 12 and Figure 13 where both the elites and commoners exist in the society but they both consume equally implying that $x_e \neq 0$ and κ = 1. Similar results are seen as in an Egalitarian Society at different values of δ, where a collapse is observed at higher values and a steady state at lower values. But in another case when κ = 100 (Figure 15) which is the case in an Unequal Society in the paper, a collapse is observed after an apparent equilibrium due to high inequality in the society. Whereas even in an Unequal Society with lower value of κ = 10 (Figure 14) a steady state is reached on a long run.

DIFFERENT SCENARIOS FROM “HANDY”

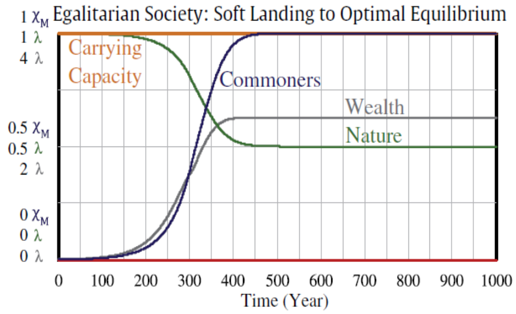

Figure 10. A soft landing to the optimal equilibrium is observed in the figure where the elite population in red is equal to zero and the final population reaches the maximum carrying capacity at a low value of the rate of depletion of nature $\delta = 6.67\times10^{-6}$ Adapted from Motesharrei et al., 2014.

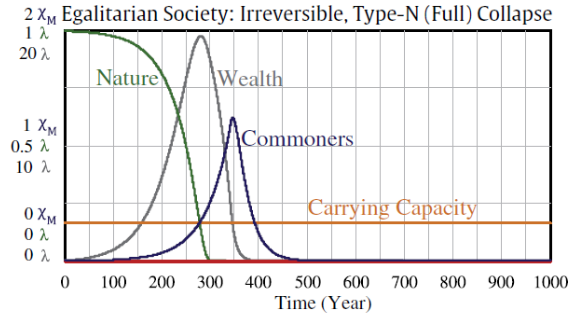

Figure 11. An irreversible collapse is observed in the figure where again the elite population in red is equal to zero in an egalitarian society. All the state variables collapse to zero in this scenario due to over depletion of nature at a higher value of the rate of depletion of nature $\delta = 36.685\times10^{-6}$. Adapted from Motesharrei et al., 2014.

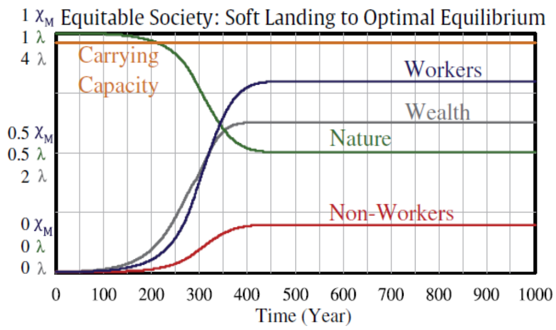

Figure 12. An equilibrium between both Workers(Commoners) and Non-Workers(Elites) can be attained with a low value of the rate of depletion of nature $\delta = 8.33 \times 10^{-6}$. Adapted from Motesharrei et al., 2014, 97.

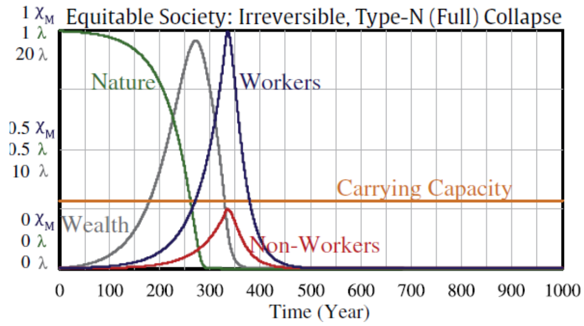

Figure 13. Whereas an Irreversible collapse in the society is observed at a higher value of the rate of depletion of nature $\delta = 4.33 \times 10^{-5}$.due to over depletion of nature. Adapted from Motesharrei et al., 2014, 97.

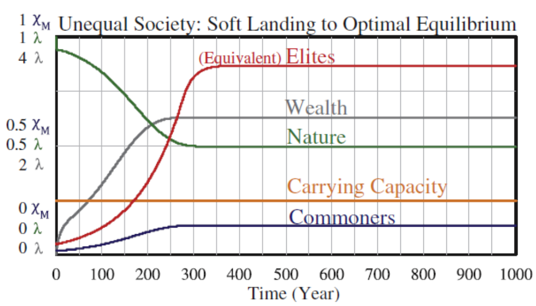

Figure 14. With a moderate inequality c, the states in the model reach an optimal equilibrium at a relatively lower depletion rate of nature $\delta = 6.35 \times 10^{-6}$ and $x_e = 3 \times 10^{+3}$ Adapted from Motesharrei et al., 2014, 98.

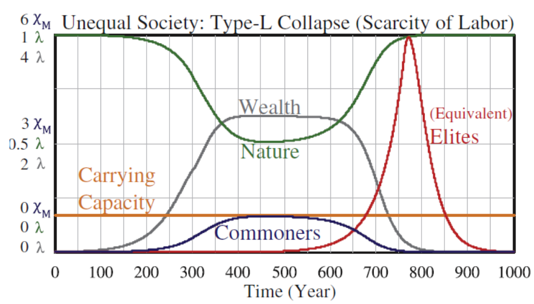

Figure 15. Population Collapse following an apparent equilibrium due to a small initial Elite population when κ =100 at a nature depletion value of $\delta = 6.67 \times 10^{-6}$ and $x_e = 1 \times 10^{-3}$ Adapted from Motesharrei et al., 2014, 98.

3.2.4.3. Summary

The model tries to highlight the history where the collapse of even advanced civilizations has occurred over the past which was followed by centuries of population and cultural decline and economic regression. The paper tries to explain this by building a simple mathematical model to explore the essential dynamics of interaction between population and natural resources. It also highlights the role inequality in a society plays towards a sustainable future. The Human and Nature Dynamics (“HANDY”) was inspired by the predator-prey model, with the human population acting as the predator and nature the prey. To sum up the findings in the paper the 2 critical features that were apparent in the historical societal collapses – over-exploitation of natural resources and strong economic stratification can both independently lead to a collapse of the society.

3.2.5. A Monetary Minsky Model of The Great Moderation and The Great Recession

Hyman Minsky was an American economist, a professor of economics who developed the financial instability hypothesis to address if the great depression could happen again and if “It” can happen, why didn’t “It” occur in the years since World War II? He advocated that to answer these questions it is necessary to have an economic theory which makes great depressions one of the possible states in which our type of capitalist economy can find itself. (Minsky,1982a, pg.5). Minsky instead combined insights from Schumpeter, Fisher, and Keynes to develop a theory of financially driven business cycles which can lead to an eventual debt-deflationary crisis. Steve Keen in his 1995 paper titled “Finance and economic breakdown: modeling Minsky’s “financial instability hypothesis"[28] highlights that Minsky's attempts to devise a mathematical model of his hypothesis were unsuccessful (Minsk, 1957)[29] arguably because the foundation he used the multiplier-accelerator- model was itself flawed (Keen, 2000, pg.84-89). Keen 1995 (Keen, 1995, pg. 614-618) instead used Goodwin's growth cycle model (Goodwin, 1967), which generates a trade cycle with growth out of a simple deterministic structural model of the economy as described earlier in section 3.2.3.

1. State Variables
    a. Output (Y)
    b. Rate of change of real wages (w)
    c. Debt (D)
    d. Labor Productivity (a)
    e. Population (N)
2. State equations
    a. Rate of change of Output

$$\frac{dY}{dt} = Y\left(\frac{I\left(\frac{\Pi}{vY}\right)}{v} - \gamma\right) \qquad \text{Eq 45}$$

    b. Rate of change of real wage rate w via Phillips curve

$$\frac{dw}{dt} = ph(\lambda)w \qquad \text{Eq 46}$$

    c. Rate of change of Debt

$$\frac{dD}{dt} = I\left(\frac{\Pi}{vY}\right)Y - \prod \qquad \text{Eq 47}$$

---

28 https://www.jstor.org/stable/4538470?seq=1#page_scan_tab_contents

29 The first of Minsky's two papers in the AER set out a mathematical model of a financially driven trade cycle developed during his Ph.D., but he never attempted to develop the model further (Minsky,1957).

d. Rate of change of Labor Productivity a at a constant rate of growth α

$$\frac{da}{dt} = \alpha a \qquad \text{Eq 48}$$

e. Rate of change in population N at a constant rate of growth β

$$\frac{dN}{dt} = \beta N \qquad \text{Eq 49}$$

SYSTEM INTERACTIONS IN KEEN 2013, "GOODWIN WITH DEBT" MODEL

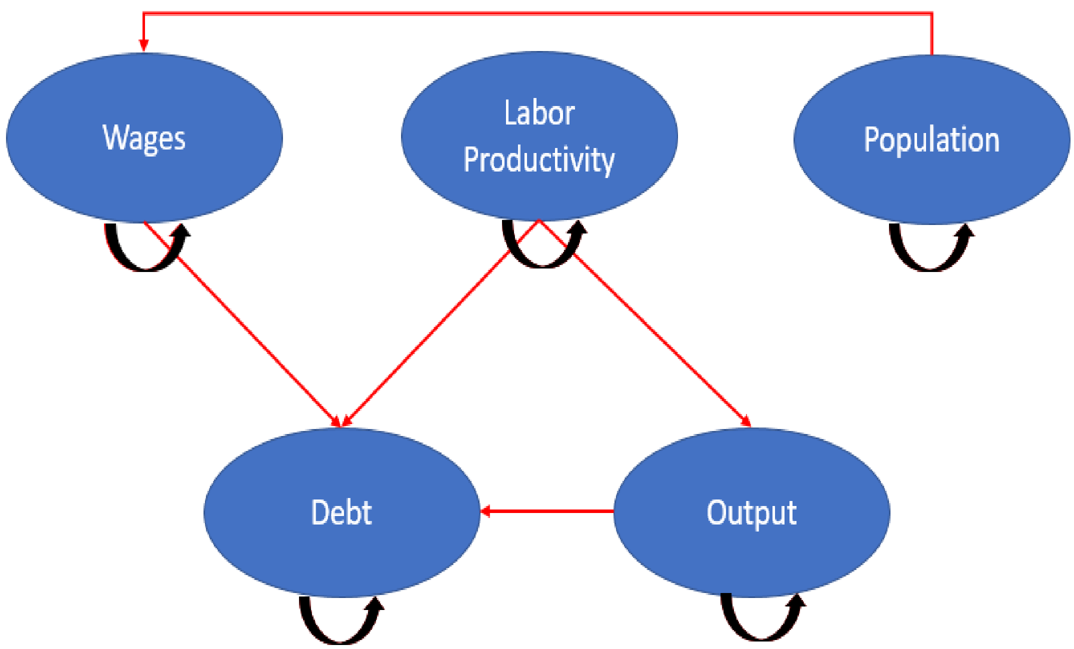

Figure 16. The above diagram shows the different interactions of different parameters in the model.

One of the modifications made by Keen in modeling Minsky's insights considering the role of debt finance was to replace the what he calls the unrealistic linear function for investment where all profits are invested in a linear relationship of Investments = Profits with a nonlinear relation as in Eq 51, so that when the desire to invest exceeds retained earnings, firms will borrow to finance investment. An exponential function for the propensity to invest captures the most fundamental of Keynes's insights about the behavior of agents under uncertainty. Thus, desired investment exceeds profits at high rates of profit, and is less than profit at low rates, because agents extrapolate current conditions into the future. Similarly, with a nonlinear Phillips curve as in Eq 52, wages rise rapidly at high levels of employment and fall slowly at lower levels.

Both the non-linear investment function (Figure 17) and the non-linear Phillips curve (Figure 18) is obtained by substituting values defined in the Eq 51 and Eq 52 below into the general expression format defined in (Keen, A Monetary Minsky Model of the Great Moderation and the Great Recession 2013, 225) Eq 50.

$$GenExp(x, x_{val}, y_{val}, s, min) = (y_{val} - \mathrm{min})e^{\left(\frac{s}{(y_{val} - \mathrm{min})}\right)(x - x_{val})} + min \quad \text{Eq 50}$$

Investment Function $I(\pi_r) = GenExp(\pi_r, 0.05, 0.05, 1.75, 0)$ Eq 51

Non-Linear Phillips curve $P_h(\lambda) = GenExp(\lambda, 0.95, 0.0, 0.5, -0.01)$ Eq 52

NON-LINEAR INVESTMENT FUNCTION AND PHILLIPS CURVE DEFINED BY KEEN IN KEEN, 2013

Investment Function

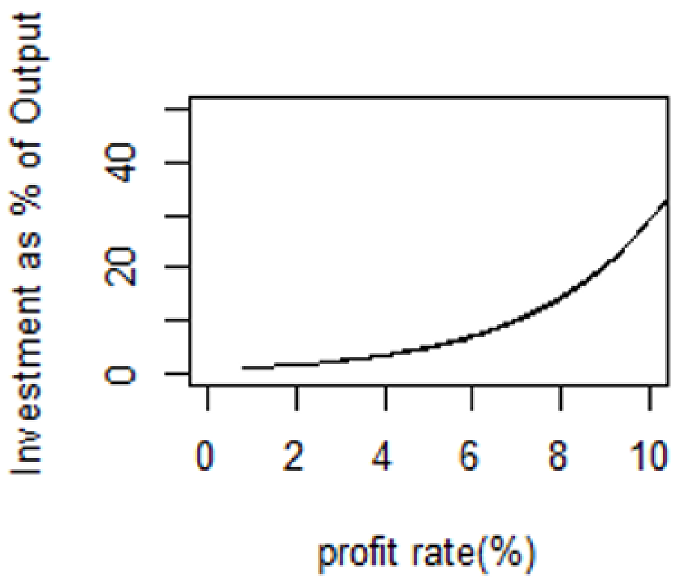

Figure 17. The level of Investment in GDP as a function of the rate of profit.

Non-Linear Phillips curve

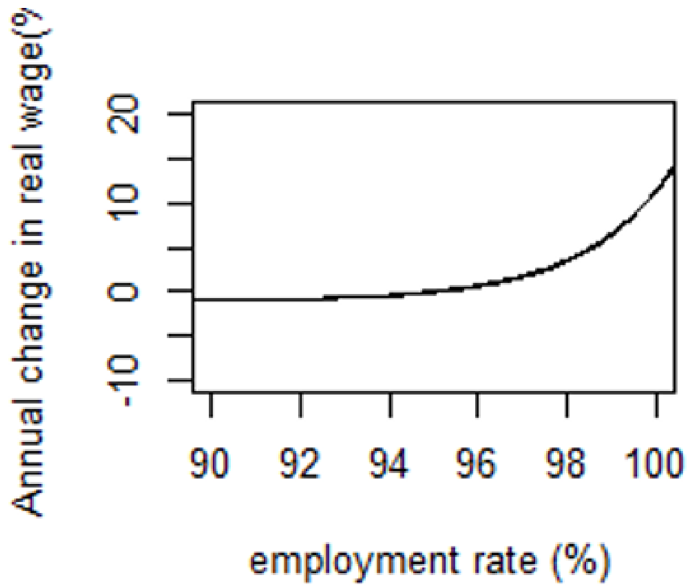

Figure 18. The rate of change of real wages as a function of the employment rate.

GOODWIN MODEL WITH NON-LINEAR FORM FOR PHILLIPS CURVE WITH DEBT (FIGURE 19) AND NO-DEBT (FIGURE 20)

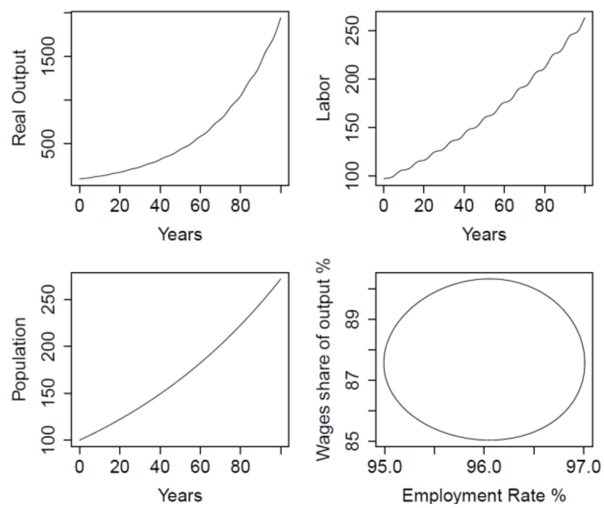

Figure 19. Goodwin model exponential Phillips curve with No-debt.

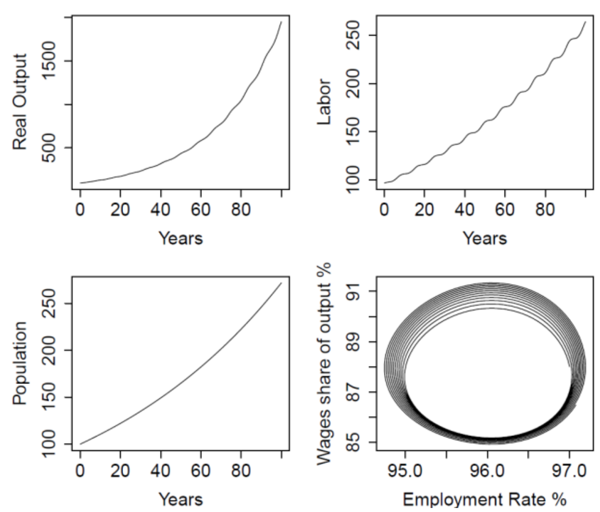

Figure 20. Goodwin model exponential Phillips curve with debt.

4. CHAPTER 4: RESEARCH DESIGN

4.1. SCOPE OF RESEARCH

The scope of the research will be to merge the models (Table 1) discussed so far and while doing so maintain consistency in order to answer the research question in the most effective way. The Biophysical model "HANDY" (Motesharrei, Rivas and Kalnay 2014) will be merged with the Goodwin model with Debt and No-Debt as described in Keen, 2013. In the "HANDY" model everything eventually comes from nature, the commoners extract nature which in return gets stored as accumulated wealth. This wealth is then consumed by both the elites and the commoners. The economic model described by Keen considers an exponential growth of labor productivity at a constant rate α (Eq 48) and thus determining the output according to the Eq 22. It also considers an exponential growth for population (Eq 49) unlike in "HANDY" where the population (Eq 35) is the difference between the births and deaths and death rate being a function of total accumulated wealth (Eq 39 & Eq 42).

Assumptions in the merged model:

1. There is only one kind of population as defined in "HANDY" (Eq 35) in our merged model which will be categorized into 2 kinds:
    a. Employed Labor (the part of the population that is employed), and
    b. Unemployed Labor (the population that isn't part of the labor that contributes to the growth of the economy)
2. When the rate of extraction (depletion) of nature is assumed to need labor as an input, the merged model explicitly considers only employed peoples as labor, whereas in HANDY the entire commoner population,

with no concept of employment is assumed as working to extract the nature in order to accumulate wealth.

3. Labor is defined in our model under the assumption of full utilization of capital in the Leontief production function and can be calculated from the equation $Y = aL = K/v$ similar as in Keen (2013).
4. A constant rate of interest is assumed on the debt accumulated towards investment.

Table 1. Equations for the merged model

"HANDY" Equations (Motesharrei et. al, 2014)	**Keen, 2013 Equations**
1. $\frac{d}{dt}x_c = \beta_c x_c - \alpha_c x_c$ *; Eq 35* *2.* $\frac{d}{dt}x_e = \beta_e x_e - \alpha_e x_e$ *; Eq 36* *3.* $\frac{d}{dt}y = \gamma\, y\,(\lambda - y) - \delta x_c y$ *; Eq 37* *4.* $\frac{d}{dt}w = \delta\, x_c y - C_c - C_e$ *; Eq 38*	*1.* $\frac{dY}{dt} = Y(\frac{I\left(\frac{\Pi}{vY}\right)}{v} - \gamma)$*; Eq 45* *2.* $\frac{dw}{dt} = ph(\lambda)w$*; Eq 46* *3.* $\frac{dD}{dt} = I\left(\frac{\Pi}{vY}\right)Y - \prod$ *; Eq 47* *4.* $\frac{da}{dt} = \alpha a$*; Eq 48* *5.* $\frac{dN}{dt} = \beta N$*; Eq 49*
Merged Equations	
1. $\frac{dx_{hc}}{dt} = \beta_{hc}x_{hc} - \alpha_{hc}x_{hc};$ Eq 53 2. $\frac{dy_h}{dt} = \gamma_h y_h(\lambda_h - y_h) - \delta L y_h;$ Eq 54 3. $\frac{dw_h}{dt} = \delta L y_h - C_{hc};$ Eq 55 4. $\frac{dY}{dt} = Y\left(\frac{I\left(\frac{\Pi}{vY}\right)}{v} - \gamma\right);$ Eq 56 5. $\frac{dw}{dt} = ph\,(\lambda)w;$ Eq 57 6. $\frac{da}{dt} = \alpha a;$ Eq 58 7. $\frac{dD}{dt} = I\left(\frac{\Pi}{vY}\right)Y - \prod;$ Eq 59	

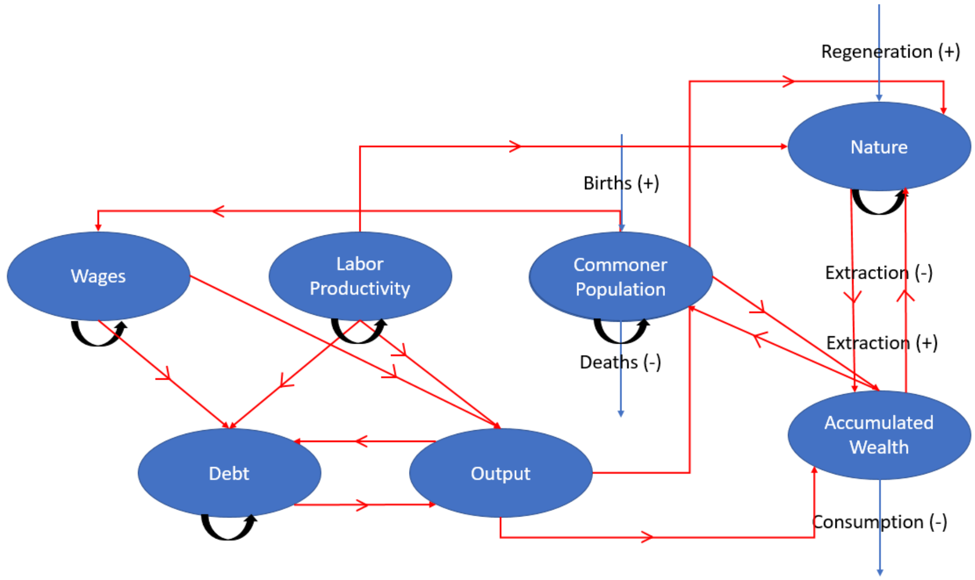

Figure 21. The above diagram explains the how the 2 models have been integrated to understand the dynamics of the merged model.

The concepts of Output (Y), Labor (L) and Debt (D) and exponential growth in labor productivity (a) will be used to understand their feedback on biophysical parameters like population and nature in "HANDY". Thus, modifying the nature extraction (depletion) function and the accumulated wealth in the Handy model to that as above (Eq 54 & Eq 55 respectively)

In this chapter, I introduce alternative functions to describe nature extraction (depletion) function in the merged model to understand the implications of different assumptions. The different types of nature extraction (depletion) function are:

a. Nature extraction (depletion) as a function of Labor (L)

$$Nature\ Extraction = \delta_L L y_h \quad \text{Eq 60}$$

b. Nature extraction (depletion) as a function of Capital (K)

$$Nature\ Extraction = \delta_K K y_h \quad \text{Eq 61}$$

c. Nature extraction (depletion) as a function of Power Input (Pi)

We define Power Input (Pi) as a portion of wealth denoted by $\delta_{Pi} wealth^{-1} time^{-1}$ towards providing power to extract nature and therefore,

$$Nature\ Extraction = \delta_{Pi} Piy_h \qquad \text{Eq 62}$$

d. Depreciation of Wealth

The concept of depreciation was missing for accumulated wealth in HANDY and therefore we will have it in our model to understand its impact. Depreciation of wealth in our model is represented by δ_h which has a constant value of 1%/yr. and modifies the wealth equations as below:

$$\frac{d}{dt} w_h = \delta_L L y_h - C_{hc} - \delta_h w_h \qquad \text{Eq 63}$$

$$\frac{d}{dt} w_h = \delta_K K y_h - C_{hc} - \delta_h w_h \qquad \text{Eq 64}$$

$$\frac{d}{dt} w_h = \delta_{Pi} Piy_h - C_{hc} - \delta_h w_h \qquad \text{Eq 65}$$

4.2. OVERVIEW OF METHODOLOGY

There are a couple of ways to merge the two models while either keeping the parameters the same or altering them to help us understand the research question. Most of the parameters in both the models have been kept the same except a few listed below in Figure 22.The predominant idea is to try out different forms for the Nature Extraction (depletion) function but there are other parameters that affect the models and it’s important to understand their feedbacks to use the best approach

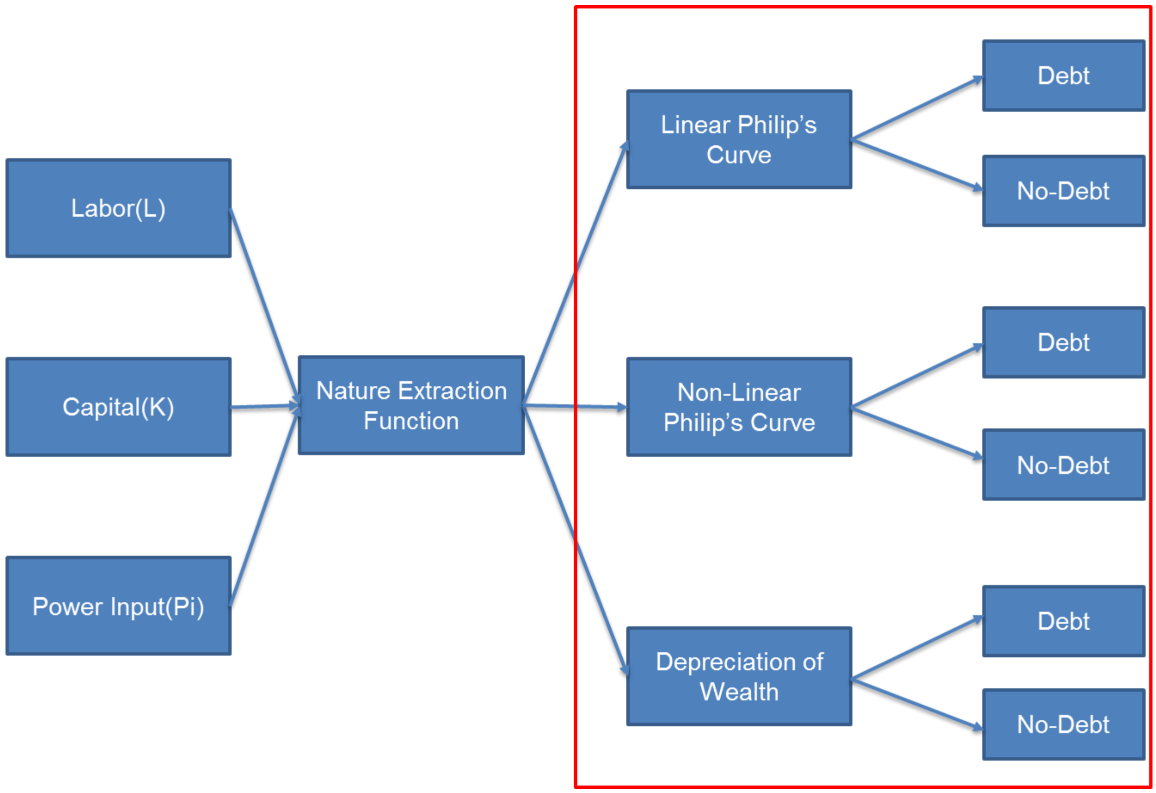

Figure 22. Different parameters from both the models which will be tweaked in the merged model to understand how sensitive the model is to these parameters.

4.3. Research Goal

The research goal of the study here would be to identify the best scenario after simulating the different scenarios discussed above in section 4.2. and try to understand it in detail and confirm if the results are consistent with the other two models.

5. CHAPTER 5: RESULTS AND FINDINGS

5.1. NATURE EXTRACTION AS A FUNCTION OF LABOR

In this section, we assume that only labor (L) is used to extract nature which gets stored as wealth and is consumed by the total population in the system (both the employed and the unemployed population in the model).

5.1.1. NATURE EXTRACTION AS A FUNCTION OF LABOR NOT INCLUDING DEBT AND A LINEAR FORM FOR THE PHILLIPS CURVE

I model the merged equations from the 2 models ("HANDY" + "Goodwin + Debt") as in Table 1 without debt, and using a linear form for the Phillips curve equation $ph(\lambda) = -C + d\lambda$ as seen in Eq 26. Figure 23 shows the results of simulating the equations with a baseline extraction rate $\delta_{L,o} = 6.67\times10^{-6}$. After simulating the model for 500 years the accumulated wealth, available nature, and population reaching a steady state between 300 to 400 years. The output grows continuously through the entire period with declining profits. The profit rate remains positive during the entire simulation. The profit rate (Keen, 1995, 616) in the model fluctuates in the range of 0.03 to 0.08 for ~ 200 years and declines as wage share increases.

Without Debt,

$$Profit\ Rate = \frac{1}{v}\left(1 - \frac{w}{a}\right) \qquad \text{Eq 66}$$

With Debt,

$$Profit\ Rate = \frac{1}{v}\left(1 - \frac{w}{a} - \frac{rD}{Y}\right) \qquad \text{Eq 67}$$

Figure A-1 shows results when the extraction rate is changed from 0.5 to 5 times the baseline extraction rate $\delta_{L,o}$. Higher extraction rates $\delta_L > 2.5\ \delta_{L,o}$ causes a more rapid growth in population and output, but eventually nature is depleted (to very near zero) and most wealth is consumed such that the population crashes after reaching its peak. Nature doesn’t accumulate to a significant degree that total consumption is small enough for nature to grow back. The higher the extraction rate, the further nature is depleted, the longer nature takes to accumulate, and thus the closer population declines to zero.

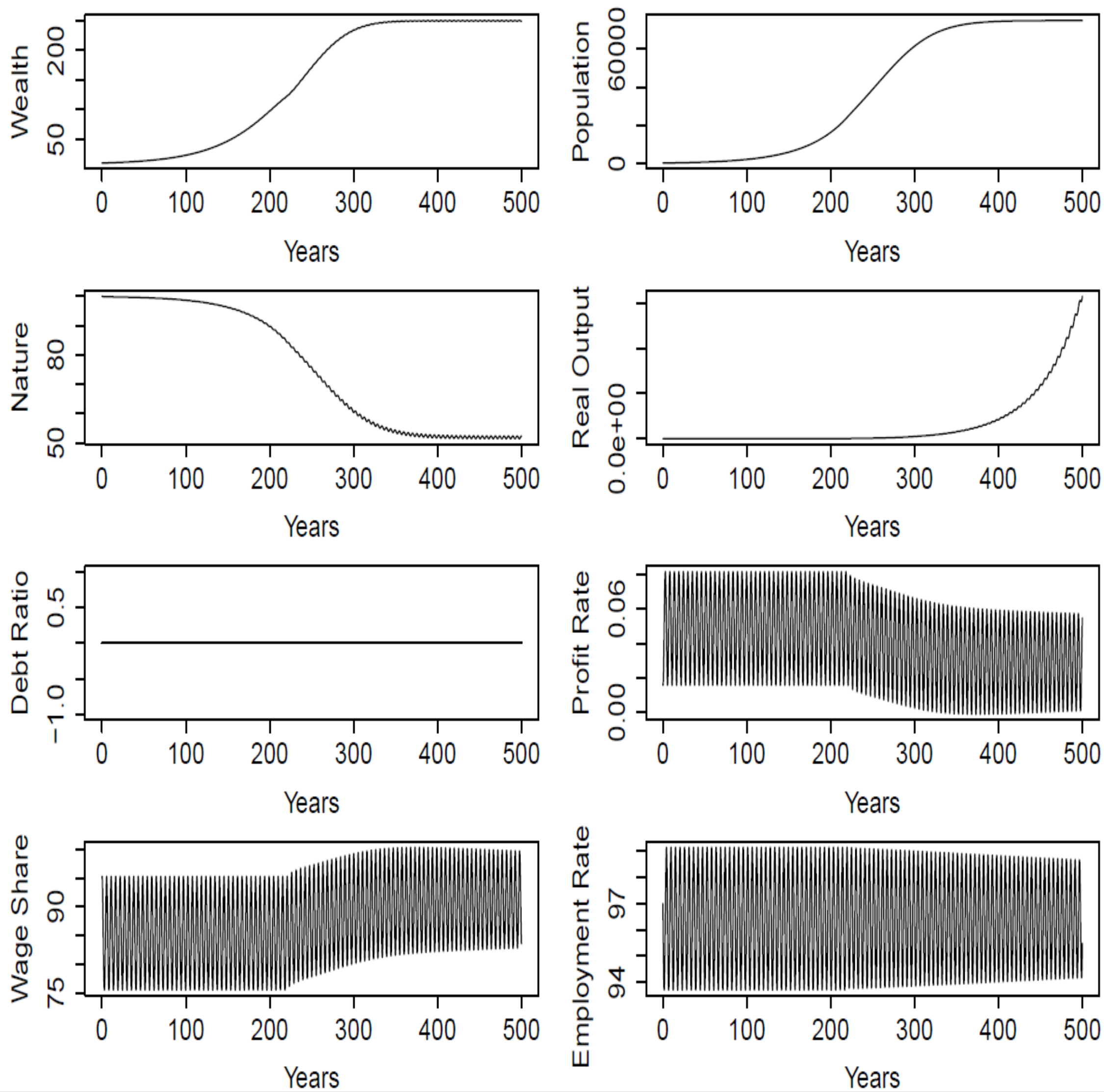

Figure 23. Simulation result when extraction rate of nature is only a function of Labor, and a baseline rate of extraction $\delta_{L,o} = 6.67\times10^{-6} person^{-1} time^{-1}$. Wages being modeled according to a linear form of the Phillips curve without including debt.

5.1.2. Nature extraction as a function of Labor including debt, linear Phillips curve, and a nonlinear investment curve

When Debt (Eq 47) is introduced in the above model with a constant rate of interest of 5% its simulation results can be seen in Figure 24. Unlike in the previous case that does not include debt, the system states do not reach steady state. Thus, resulting in declining population, output, and profit rate due to rising debt. The system blows up (~400 years) as the debt keeps on accumulating rapidly and beyond which the simulations for wage share and employment rate start going erratic. Figure A-3 shows the effect of varying the extraction rates, and similar observations can be made as in the previous case with no debt. An earlier collapse occurs as the extraction rate increases. Figure A-4 highlights the effect of different values for constant rate of interests. As the constant rate of interest on debt increases from 2% to 5% the system collapse occurs relatively earlier. However, for interest rates < 2% the system avoids collapse within the simulated time of 500 years but when the model is simulated for a longer period of time (Figure A-2) a collapse occurs eventually due to rising debt even at a lower interest rate.

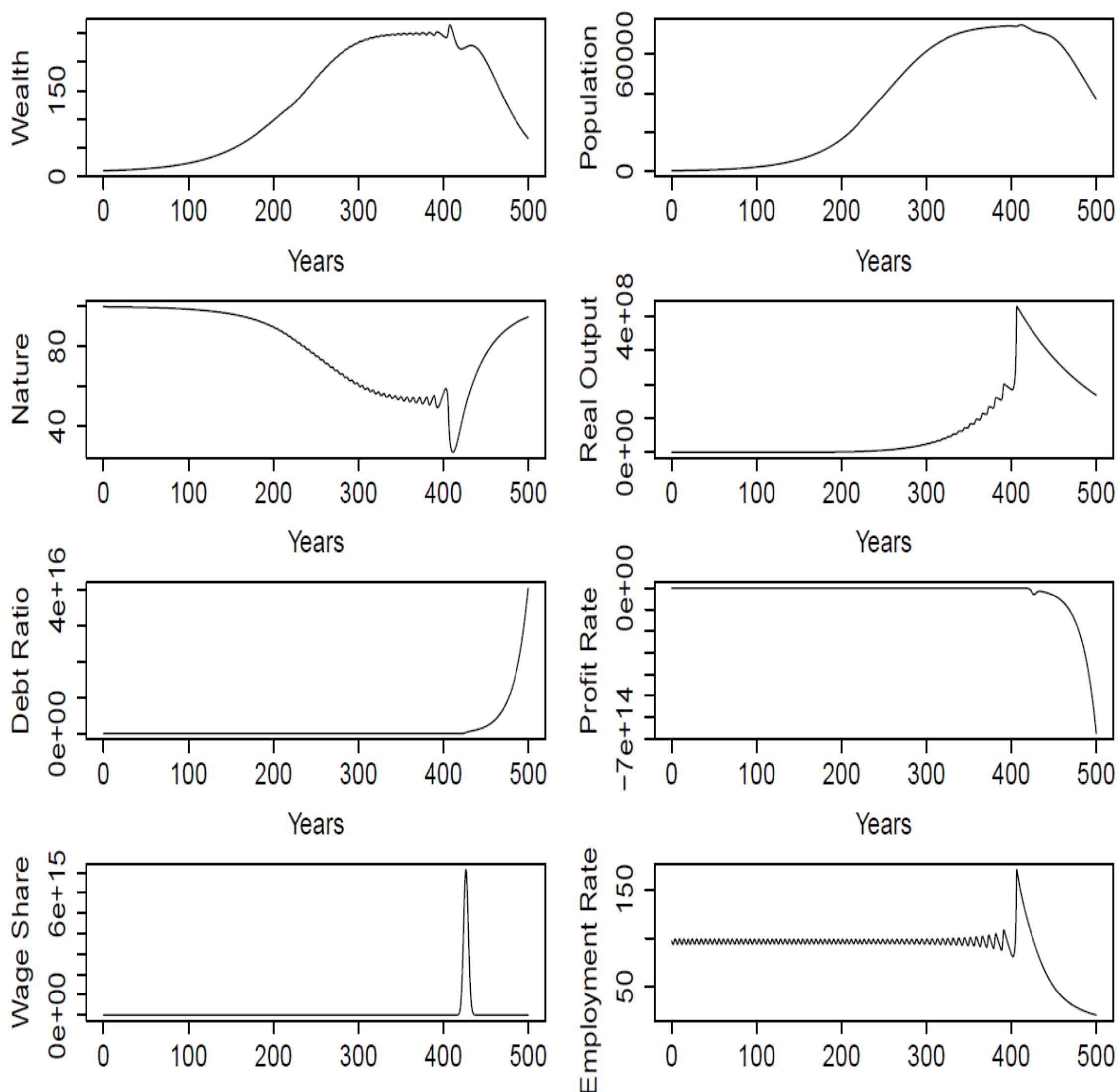

Figure 24. Simulation results for when extraction rate of nature is a function of Labor, with a baseline extraction rate $\delta_{L,o} = 6.67{\times}10^{-6}$ with debt being accumulated at a constat rate of interest $r = 5\%$ and a Linear form for the Phillips curve.

5.1.3. NATURE EXTRACTION AS A FUNCTION OF LABOR WITHOUT DEBT, NONLINEAR PHILLIPS CURVE, AND A NONLINEAR INVESTMENT CURVE

The Keen, 2013 paper suggested that a nonlinear form for the Phillips curve (Eq 52) would better the rate of change of real wages, w. Figure 25 shows the results for simulating the model with the nonlinear form for the Phillips curve with no debt in the model. The model shows similar results to as in Figure 23 at an Extraction rate of $\delta_{L,o} = 6.67\times10^{-6}$. Figure A-5 shows results when the extraction rate is varied from 0.5 to 5 times $\delta_{L,o}$ where similar results can be interpreted that, increasing extraction rate leads to an earlier collapse in the model, same as in the case with the linear form for the linear Phillips curve.

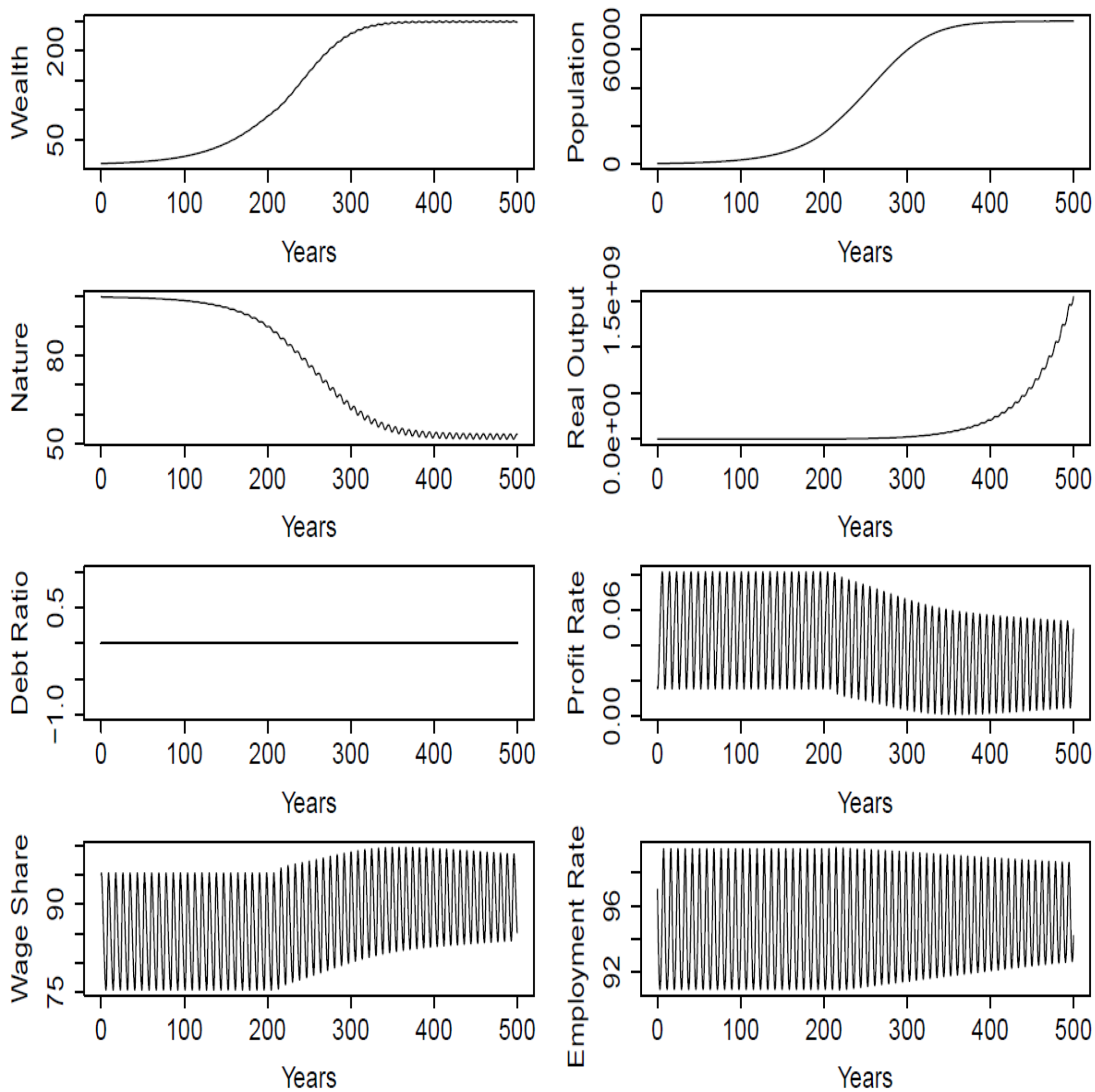

Figure 25. Simulation results for when extraction rate of nature is a function of Labor, with a baseline extraction rate $\delta_{L,o} = 6.67\times10^{-6}$ without debt and a nonlinear form for the Phillips curve.

5.1.4. NATURE EXTRACTION AS A FUNCTION OF LABOR INCLUDING DEBT NONLINEAR PHILLIPS CURVE, AND NONLINEAR INVESTMENT CURVE

When debt is included in the model with a nonlinear form for the Phillips curve a debt-induced collapse is observed (Figure 26) similar to that of when a linear Phillips curve was used in the previous section 5.1.2. Similar results to that of Figure A-3 and Figure A-4 can be seen in Figure A-6 and Figure A-7 when the rate of extraction of nature and the constant rate of interest on the debt is varied. System collapse occur earlier when increasing value of $\delta_{L,o}$ and r. Charging higher rates of interest on debt increases debt payments and leads to earlier collapse.

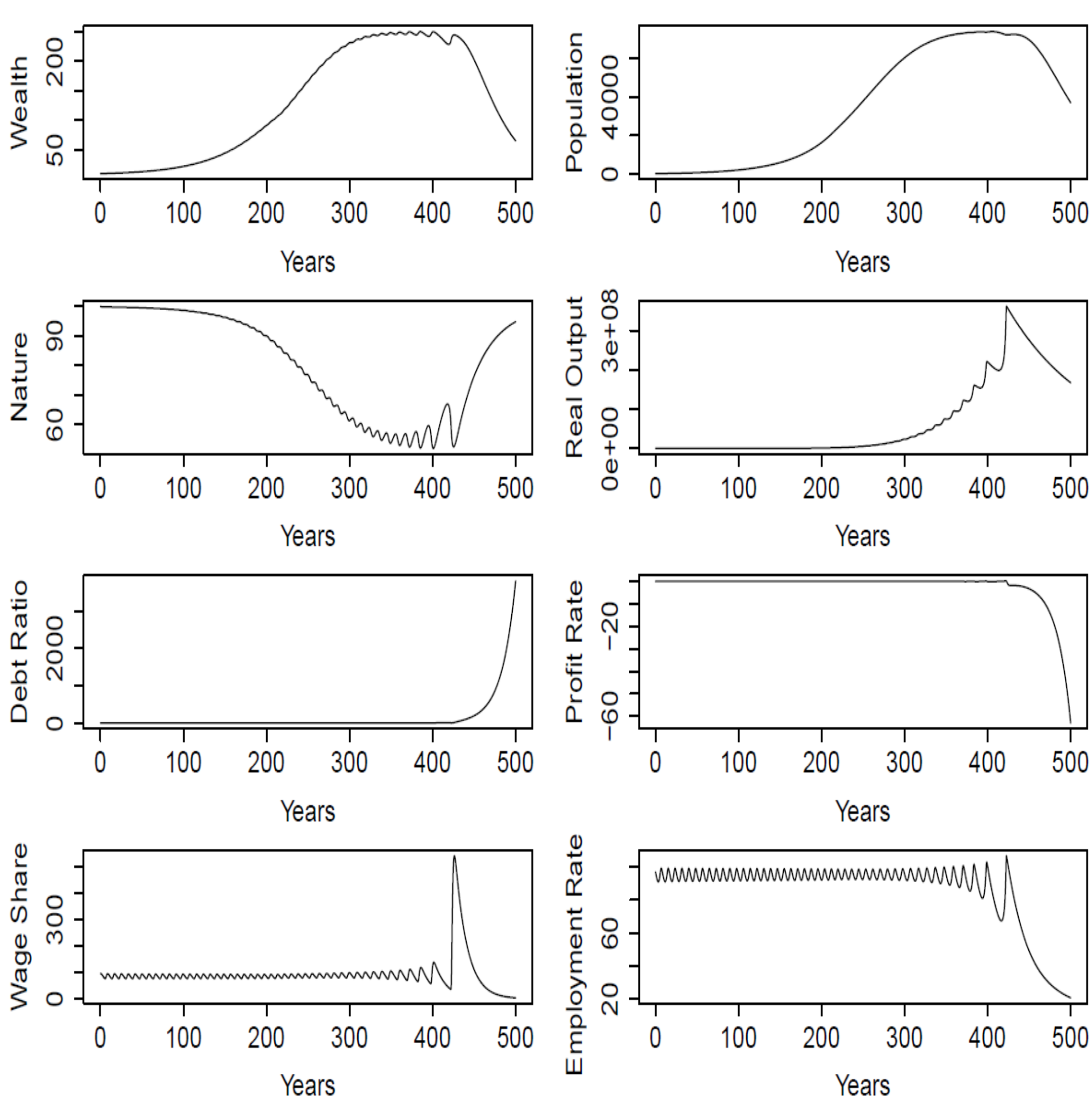

Figure 26. Simulation results for when extraction rate of nature is a function of Labor, with a baseline extraction rate $\delta_{L,o} = 6.67\times10^{-6}$ .Where debt is being accumulated at a constat rate of interest of (r) 5% and and wages modeled according to a noninear form of the Phillips curve.

5.2. NATURE EXTRACTION AS A FUNCTION OF CAPITAL

In the previous section 5.1, we had modeled the rate of extraction of nature as a function of only labor. In this section, we will model the rate of extraction of nature as a function of only capital (Eq 61). The assumption here is that the machines only (or the money needed to purchase machines) are needed to extract nature, but neither labor or wealth are explicitly needed for nature extraction. The purpose of this assumption is to explore the influence on simulation results from nature extraction being a function of capital. The baseline rate of extraction of nature is specified as $\delta_{K,o} = 3.335\times 10^{-7} capital^{-1} time^{-1}$. One very important thing to consider while going through the results in this section is that, when either the wage share or the employment rate go beyond realistic values i.e. wage share > 100% or employment rate > 100% the model behaves erratically and the results beyond that point become inconceivable and therefore should be ignored.

5.2.1. NATURE EXTRACTION AS A FUNCTION OF CAPITAL WITHOUT DEBT AND A LINEAR FORM OF THE PHILLIPS CURVE

Here we model the set of equations in Table 1 with the difference that the nature extraction is a function of only capital (Eq 61). Figure 27 shows the results of the simulation at $\delta_{K,o} = 3.335\times 10^{-7} capital^{-1} time^{-1}$ and no debt. The observations can be summarized as below:

All nature gets exhausted around 220 years without regenerating for rest of the simulation. Once nature gets exhausted the population sustains for further longer time based on the accumulated wealth during the years before nature is exhausted. Once the accumulated wealth starts declining a fall in the population and output is observed. The decline in output can be explained by a decline in labor which is itself driven by a decline in population.[30]Figure A-8 shows results when the rate of extraction of nature as a function of capital is varied from 0.5 to 5 times the baseline value $\delta_{K,o} = 3.335\times 10^{-7} capital^{-1} time^{-1}$. The major conclusion from simulating nature extraction as a function of capital is that this assumption always causes the full depletion of nature, eventually resulting in a collapse. The higher the value of δ_K, the earlier the collapse is observed in the system. Also, the profit rate here goes negative with increasing wage share according to Eq 66. As the population decreases the wage share increases, the reasoning behind this could be that the firms try to employ people by paying them higher wages until the population totally collapses.

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[30] $Output\ (Y) = a(Labor\ Productivity) * L(Labor)$

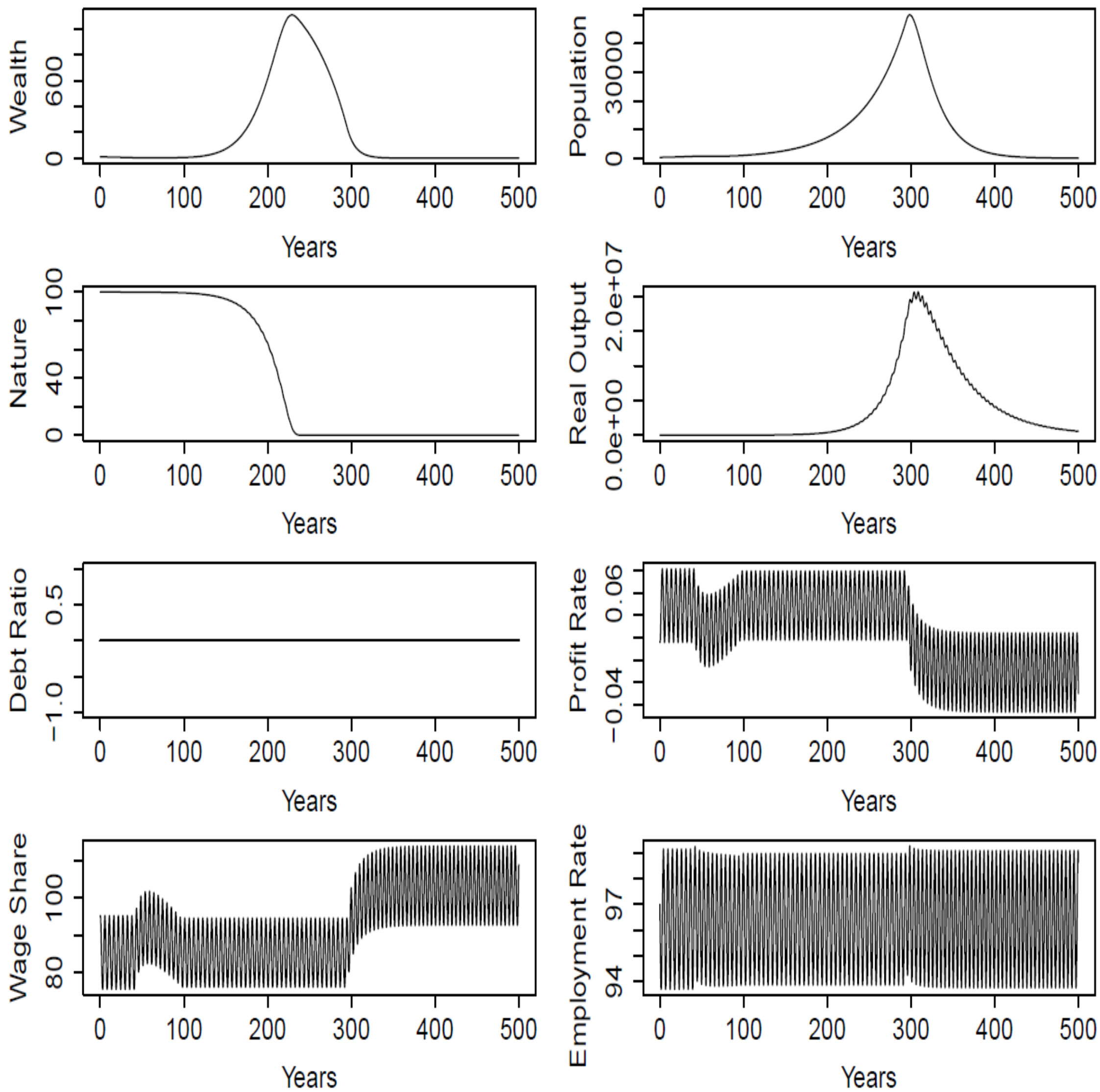

Figure 27. Simulation results for when extraction rate of nature is a function of capital, with the baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7}$ including no debt and modeling wages as a linear form of the Phillips curve.

5.2.2. Nature extraction as a function of Capital including debt and a linear form of the Phillips curve

Figure 28 describes the scenario when debt is included in the model. By including debt at 5% interest, the simulation no longer indicates the full depletion of nature under the baseline extraction rate. As the accumulated debt rises, debt payments also increase which then decreases profits. Decreasing profit results in decreasing output and with decreasing output the capital going towards nature extraction declines as well (Eq 23).

Figure A-9 shows the effect of increasing $\delta_{K,o}$ in the model. Larger values of δ_K cause earlier collapse in the system. Figure A-10 shows the effect of changing constant rate of interest. At lower values of interest rates, full extraction of nature occurs whereas, at a higher rate of interest the extraction of nature occurs slowly. This gives time for the population to grow and once the debt levels start to rise, the profit rate (Eq 67) starts to fall thus resulting in declining output. With declining output, the capital towards extraction of nature declines as well (Eq 23). Therefore, nature starts to regenerate and the population starts declining as not enough wealth is being accumulated to be consumed by the population.

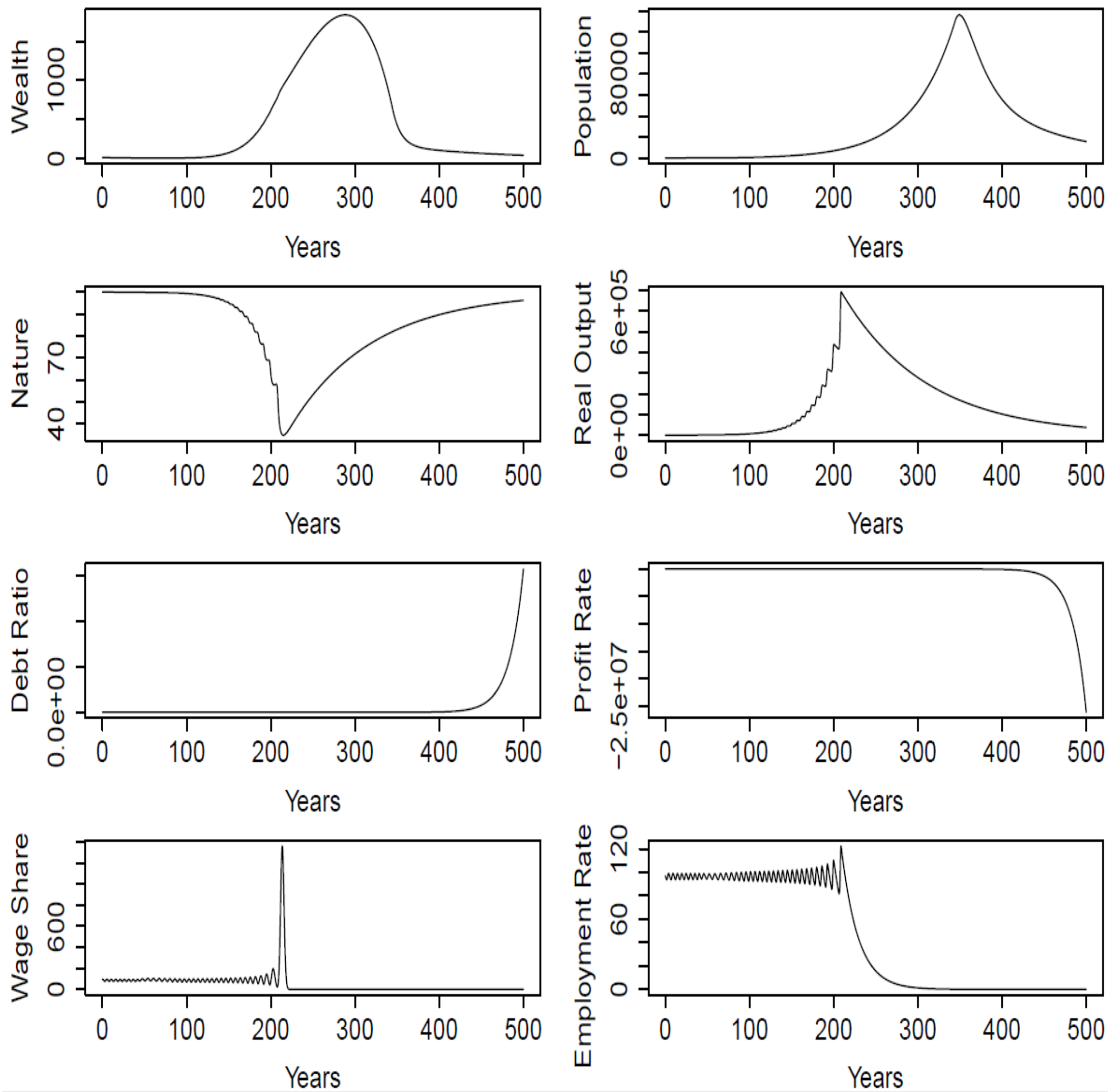

Figure 28. Simulation results for when extraction rate of nature is a function of Capital, with the baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7}$ with debt accumulating at a constant rate of interest $r = 5\%$ and wages being modeled according to the linear form of the Phillips curve.

5.2.3. Nature extraction as a function of Capital without debt and a nonlinear form of the Phillips curve

In this section, we will look at the simulation results when the wages are modeled according to the nonlinear form for the Phillips curve (Eq 52) and nature is a function of capital as in the previous section. Figure 29 shows results of the basic simulation with $\delta_{K,o} = 3.335\times10^{-7}$ and the nonlinear form for the Phillips curve. The results look similar to the results in the Figure 27, where all the nature is extracted towards accumulating wealth and not regenerating once gone because the rate of extraction is greater than the rate of regeneration under the given set of parameters. On increasing the δ_K value (Figure A-11) similar results as observed earlier in Figure A-8 can be observed where increasing δ_K causes an earlier collapse in the system.

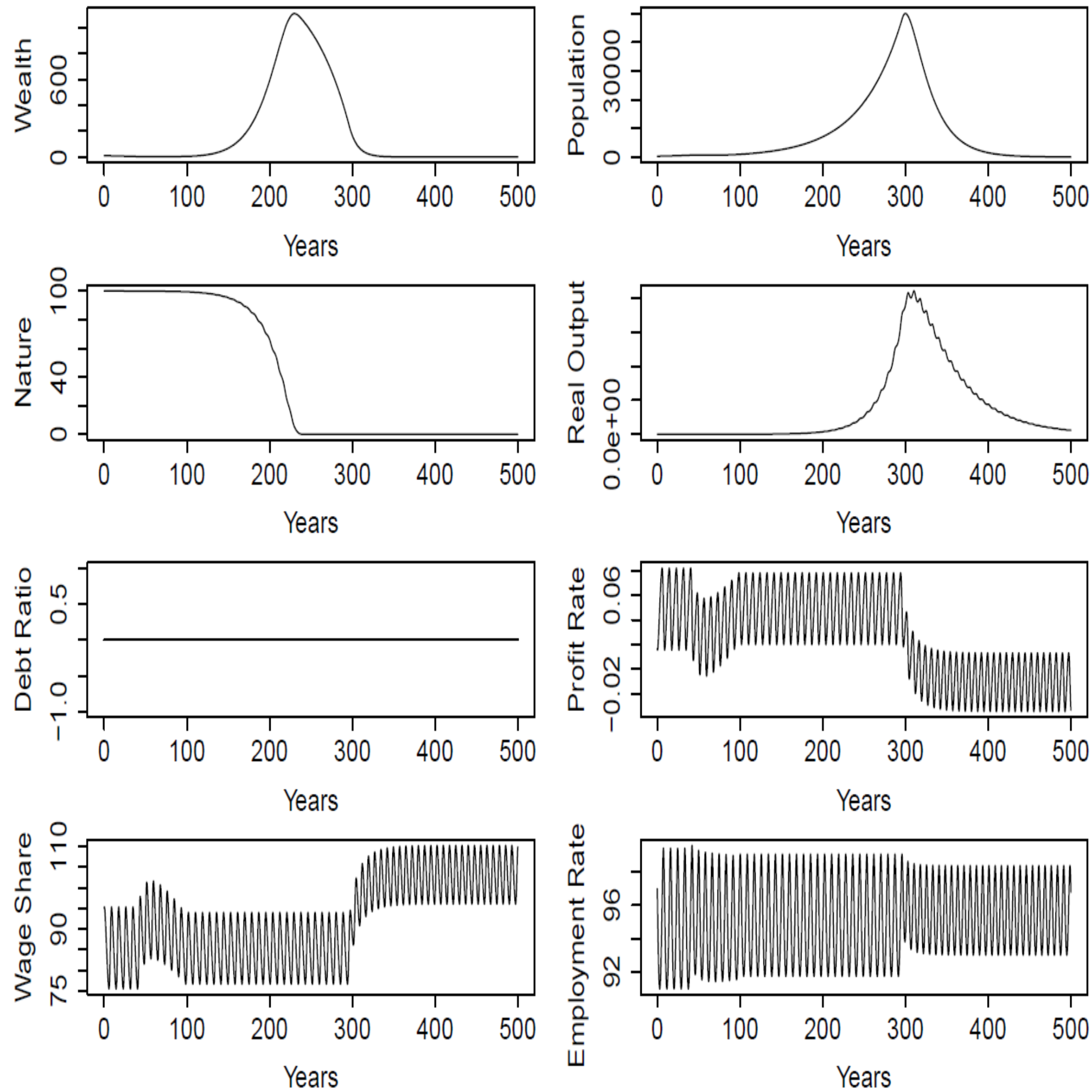

Figure 29. Simulation results for when extraction rate of nature is a function of Capital, with the baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7}$ with no debt accumulating and the nonlinear form of the Phillips curve.

5.2.4. NATURE EXTRACTION AS A FUNCTION OF CAPITAL INCLUDING DEBT, NONLINEAR FORM OF THE PHILLIPS CURVE, AND A NONLINEAR FORM OF THE INVESTMENT CURVE

In this section, we will discuss the results when simulating nature as a function of capital (Eq 61) including debt and a nonlinear form for the Phillips curve equation (Eq 52). Total Nature is still extracted in the model (Figure 30) when debt is included. The debt blows up when the wealth is totally exhausted leading to a collapse in the population after which the simulation does not provide any conceivable results (~300 years). When the δ_K is increased similar behavior as in earlier simulations is observed where the collapse occurs earlier (Figure A-12). Figure A-13 shows the results for the effect the constant rate of interest r has on the model where it is a function of only capital. Here a debt induced collapse doesn't happen like in Figure A-10 where we had the linear form for the Phillips curve but when we increase the value of r to 7% a similar collapse is observed to happen (Figure 31). The reason is that the accumulated debt when assuming a debt @5% constant rate of interest (under the given set of parameters) with the nonlinear form for the Phillips curve isn't enough to induce collapse due to debt accumulation but however a collapse is induced by depletion of nature. Even at lower interest rates the, system keeps extracting the nature until fully exhausted.

In brief, while modeling nature extraction as a function of capital, if interest rates are high enough, a debt-induced collapse occurs before nature is depleted. At low enough interest rates, collapse occurs due to depletion of nature before too much debt is accumulated. However, under no tested conditions (different interest rates or rates of extraction) can a non-zero steady state population simulated.

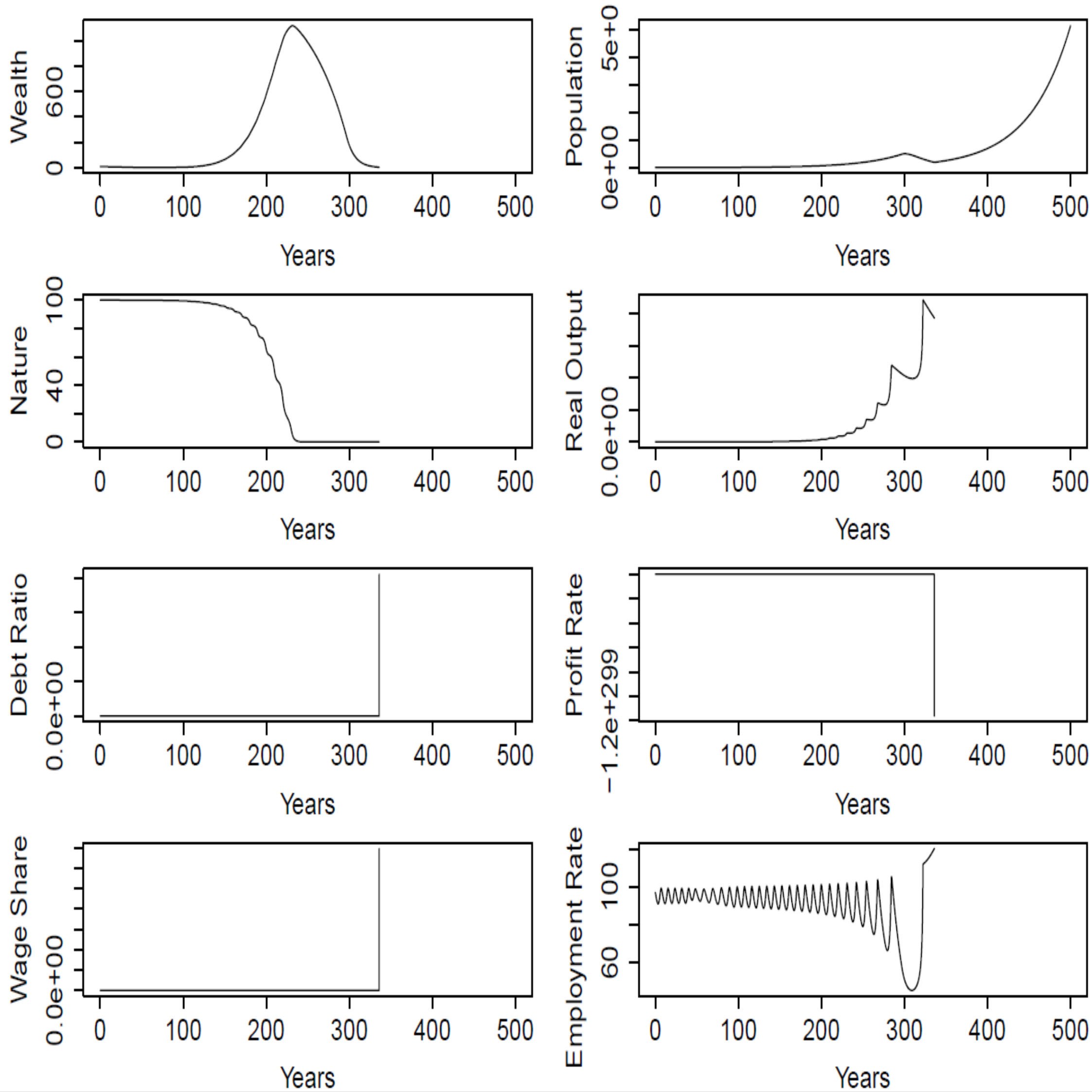

Figure 30. Simulation results for when extraction rate of nature is a function of Capital, with a baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7}$ with debt accumulating at a constant rate of interest $r = 5\%$ and a nonlinear form of the Phillips curve.

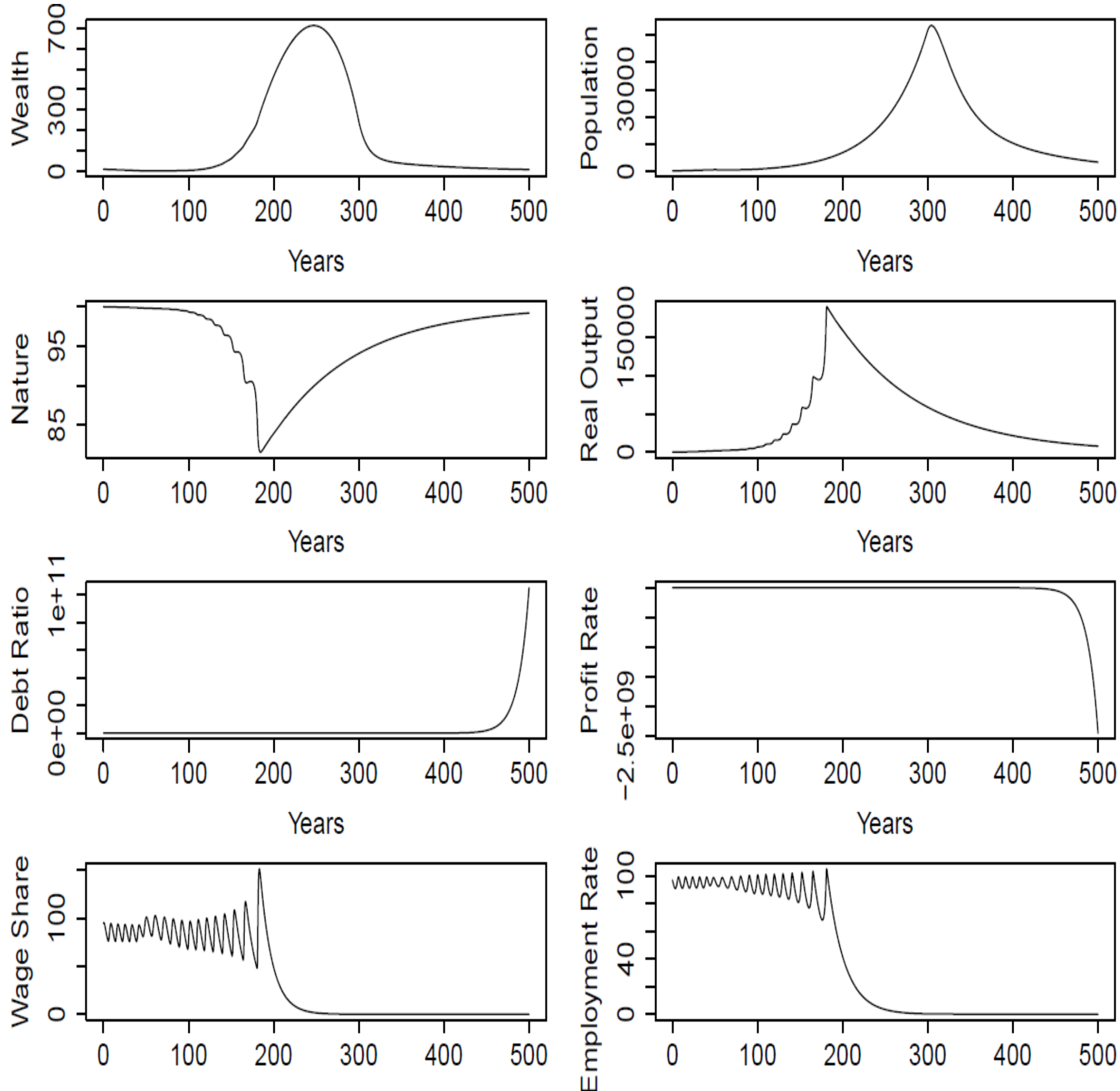

Figure 31. Simulation results for when extraction rate of nature is a function of Capital, with the baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7}$ and debt accumulating at a constant rate of interest $r = 7\%$ and Non-Linear form of the Phillips curve. A debt-induced collapse can occur if the rate of interest is high enough when the Nature Extraction is a function of Capital.

5.3. NATURE EXTRACTION AS A FUNCTION OF POWER INPUT

In section 5.1 and section 5.2 we considered nature extraction as a function of only labor and capital respectively. In this section, we will take a look at the results when the nature extraction is a function of power input. We define power input Pi (Eq 68) as the power required to extract nature which will be extracted from the stored wealth and will therefore be a function of the same. Also, in this section we will only consider wages modeled as a nonlinear function of the Phillips curve as its effect on the model has been demonstrated in the previous sections.

5.3.1. NATURE AS A FUNCTION OF POWER INPUT WITH A NONLINEAR FORM OF THE PHILLIPS CURVE WITHOUT DEBT

Here we would experiment with a new parameter defined as power input. The concept here is that the wealth stored (e.g., nature that is extracted but not yet consumed) in HANDY which initially was used only for the consumption by the population. In this section, I create an additional consumption of wealth that is the power input. The power input is defined as a fraction of accumulated wealth that is used to extract the nature. Thus, the rate of extraction of nature is now a function of how much wealth has been accumulated (Eq 62).

$$Power\ Input = pf \times w_h \qquad \text{Eq 68}$$

Where, pf is the power factor and equal to $1\%/year \times w_h$.The baseline depletion factor, in this case, is defined as $\delta_{Pi,o}$ and equal to $0.3335\ (power\ input)^{-1} time^{-1}$. Simulation results with nature extraction a function of power input can be seen in Figure 32 where nature is extracted very rapidly at the beginning but eventually reaches a steady state. The population rises at the beginning with extraction of nature but reaches a steady state at approximately the same time as nature (~200 years). Whereas, output keeps continuously growing exhibiting a scenario where exponential growth is possible in a system while attaining a sustainable population.

When δ_{Pi} is varied from 0.5 to 5 times its baseline extraction rate $\delta_{Pi,o}$ (Figure A-14) its baseline value, higher population peaks are attained at lower δ_{Pi} values in comparison to higher values for δ_{Pi}. However, reaching steady state in all cases. Whereas, output grows exponentially for different values of δ_{Pi} similar as in Figure 32 at a slower rate for higher values of δ_{Pi}.

The important result for these simulations that assume nature extraction is a function of power input (e.g., wealth consumption) is that at a low enough depletion factor, δ_{Pi}, the population and available nature can reach a steady state. At a high enough depletion factor such as $5\delta_{Pi,o}$, the population oscillates about a relatively low level.

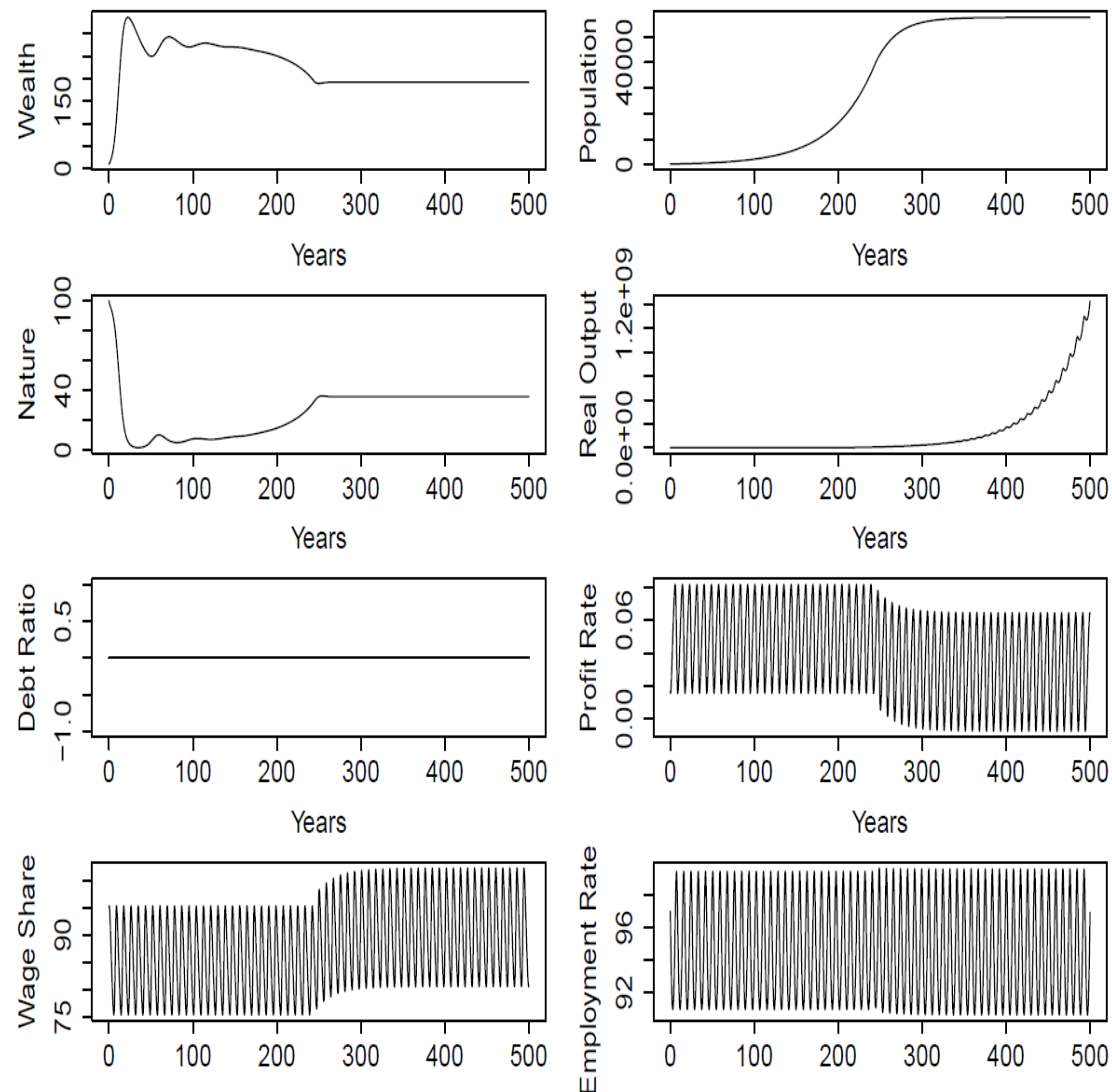

Figure 32. Simulation results for when extraction rate of nature is a function of Power Input, with a baseline extraction rate $\delta_{Pi,o} = 0.3335\ (power\ input)^{-1} time^{-1}$ with no debt and wages being modeled according to a nonlinear form of the Phillips curve.

5.3.2. Nature as a function of Power Input in the model with a nonlinear form of the Phillips curve including debt

When debt is included in the model with nature extraction a function of power input, similar results can be observed in Figure 33 and Figure A-15 to that as in the previous section with the difference of declining output and profit rate. In Figure 33, near the end of the simulation period, debt begins to increase rapidly as the population growth slows. By increasing the depletion factor δ_{Pi}, the exponential debt accumulation begins earlier as nature becomes fully depleted.

Figure A-16 highlights that there is no effect in either the population, available nature, or the output when the constant rate of interest on the debt is varied in the model. This is because there is no feedback from the economic model (Goodwin with Debt, Keen, 2013) to the parameters in the biophysical model ("HANDY'). In previous sections (5.1 and 5.2) nature extraction was a function of Labor and Capital, both these parameters were from the economic model and therefore affected the parameters in the biophysical model. Whereas here the nature extraction is a function of power input which is a function of wealth from the biophysical model. Therefore, no feedbacks are observed in the population, available nature etc.

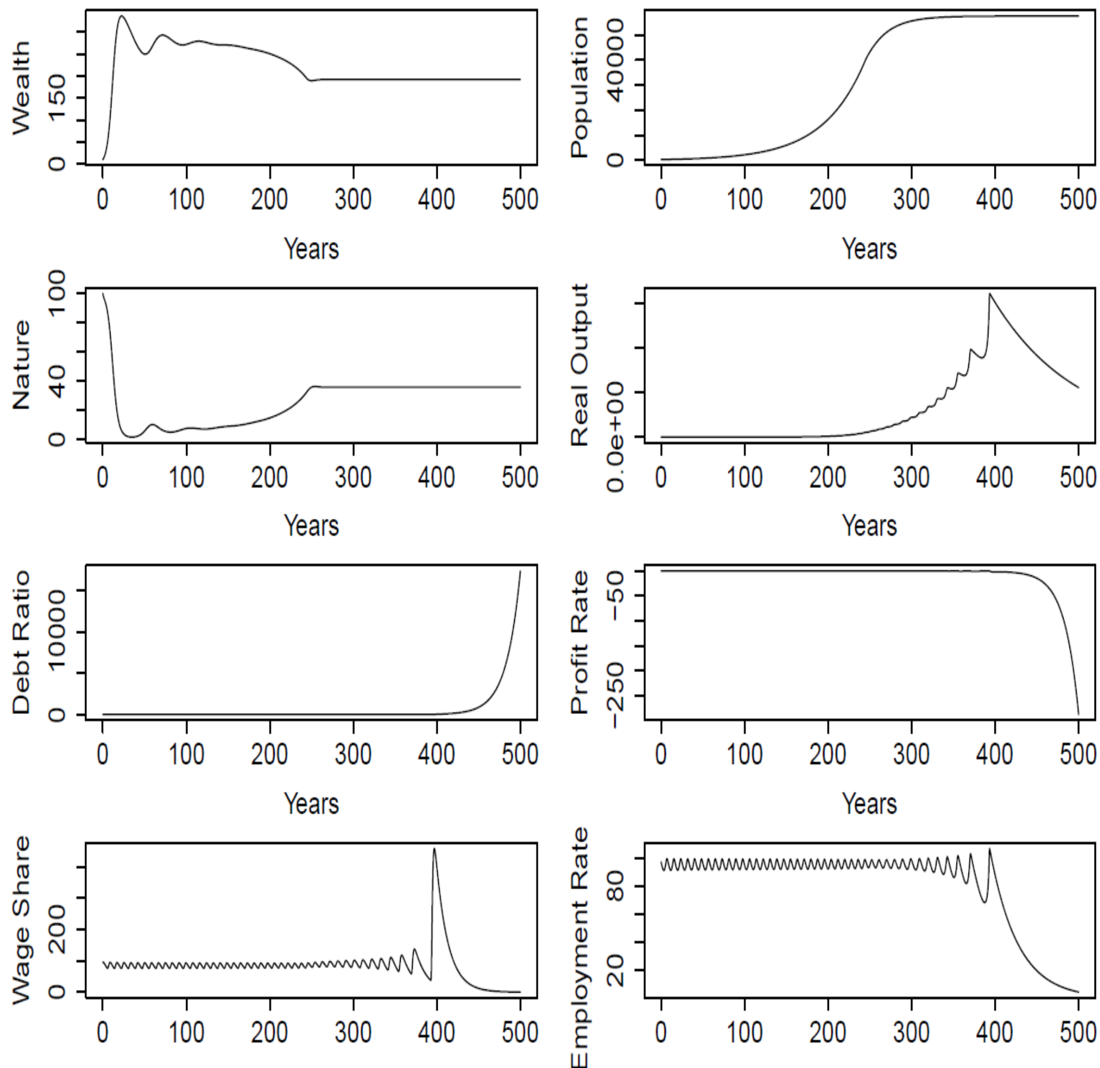

Figure 33. Simulation results for when extraction rate of nature is a function of Power Input, with the baseline extraction rate $\delta_{Pi,o} = 0.3335\ \ (power\ input)^{-1} time^{-1}$ with debt accumulating at r=5% and wages modeled according to the nonlinear form of the Phillips curve.

5.3.3. VARYING POWER FACTOR FROM 1% OF WEALTH TO 20% OF WEALTH BEING USED AS POWER INPUT TOWARDS EXTRACTING NATURE

In this section, the power factor pf is varied from 1%/yr. of wealth to 20%/yr. of wealth in order to understand its effect on the system. Figure 34 shows results when the power factor (pf) is increased from 1% to 20%. Higher values of pf result in lower peaks of the population and higher oscillations in available nature. The system seems to be reaching a steady state in all cases with higher output at lower values of pf. An interesting thing to observe is that the amount of debt accumulated is higher at higher values of pf. Implying, that the level of debt rises with decreasing population as there is less wealth to be consumed by the population and thus leading to no debt payments in the system.

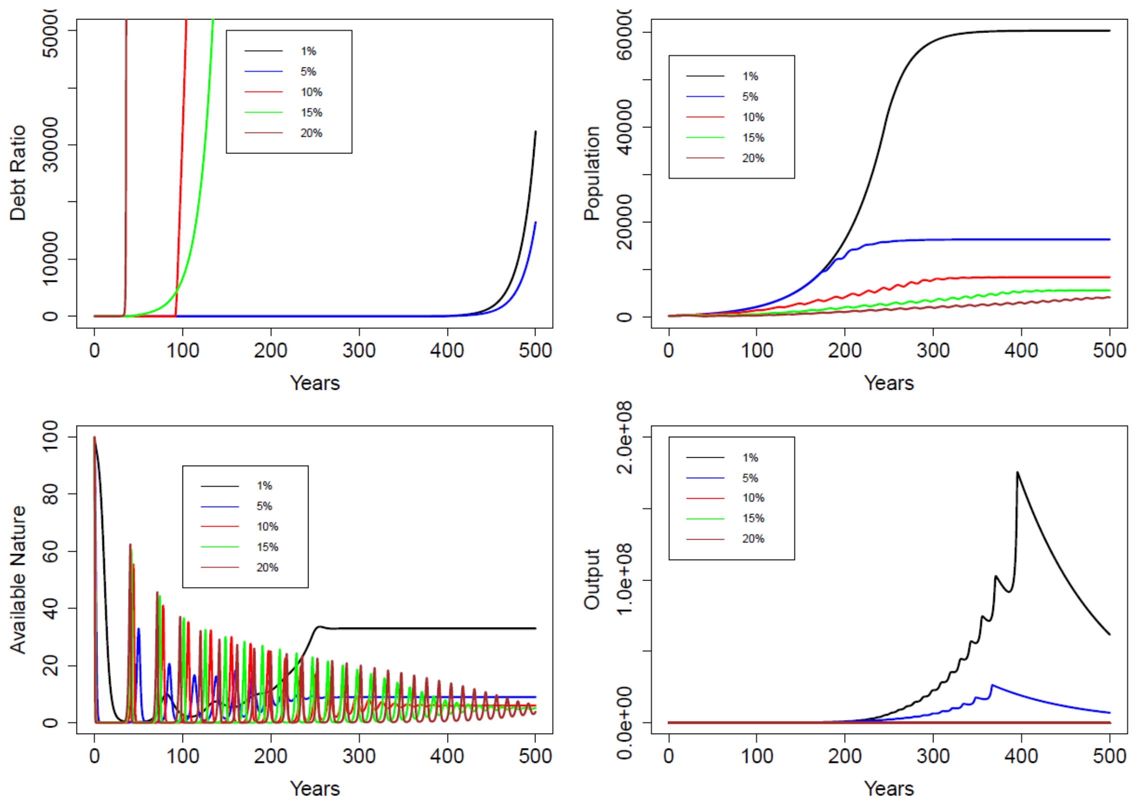

Figure 34. Simulating results by altering the power factor pf from 1% to 20% of wealth for when nature extraction is a function of Power Input, with the baseline extraction rate $\delta_{Pi,o} = 0.3335\ (power\ input)^{-1} time^{-1}$ including debt and wages modeled as a nonlinear form of the Phillips curve.

5.4. Including Depreciation of Wealth

In the earlier sections, we didn't consider any kind of depreciation in the biophysical model "HANDY". The capital depreciates at a constant rate of γ in determining the rate of change of capital K. Therefore, in this section introduce depreciation of accumulated wealth in all the 3 scenarios discussed above where

1. Accumulated wealth when nature extraction is a function of Labor Eq 63
2. Accumulated wealth when nature extraction is a function of Capital Eq 64

and,

3. Accumulated wealth when nature extraction is a function of Power Input Eq 65

Figure 35 through Figure 37 show simulated results while assuming the depreciation of wealth $\delta_h = 1\%/yr$. When compared to results with no wealth depreciation the population and wealth peaks are lower with depreciation, the population and wealth peaks are lower with depreciation. This difference is intuitive because as wealth depreciates before it can be consumed by the population, the population consume less wealth and is not able to grow as high of a peak population.

5.4.1. Comparing Results with and without depreciation of Wealth when nature extraction is a function of Labor.

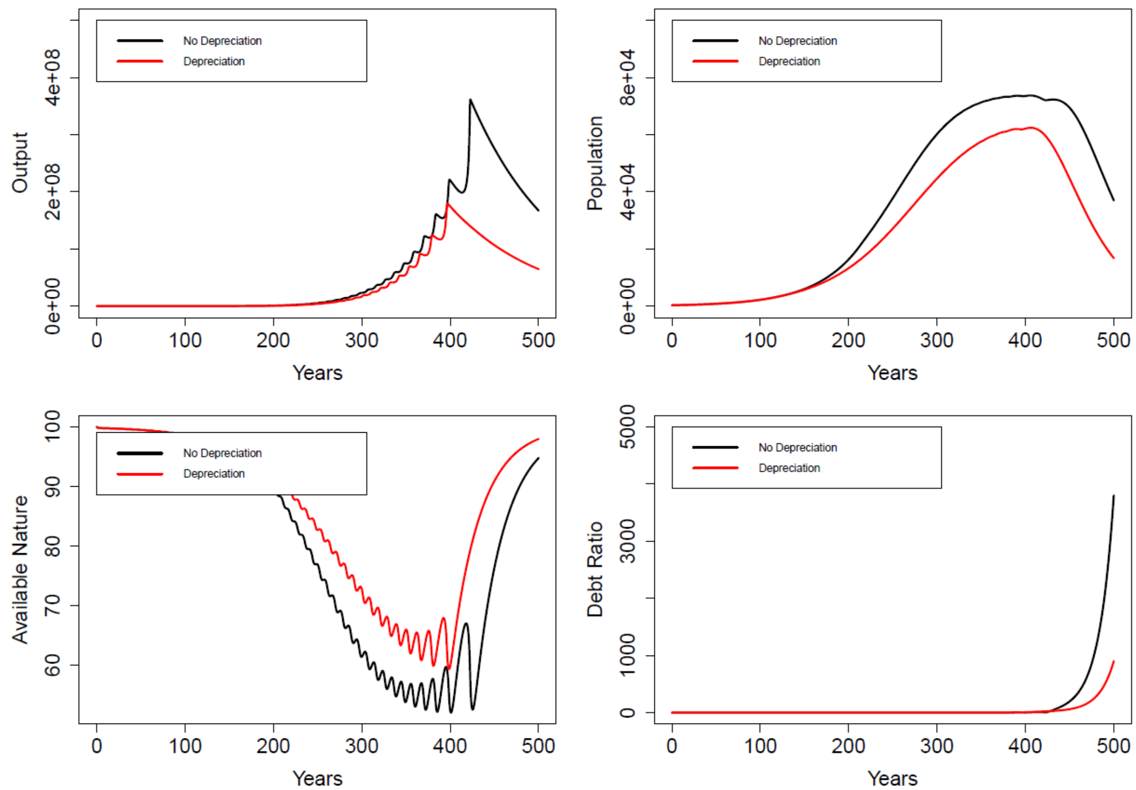

Figure 35. Simulating results for when nature is a function of labor with and without depreciation of wealth in the model. Here a nonlinear form of the Phillips curve is considered to model wages with a nonlinear form for investment including debt. The baseline extraction rate $\delta_{L,o} = 6.67\times10^{-6} person^{-1} time^{-1}$ is same as in section 5.1.

5.4.2. Comparing Results with and without Depreciation of Wealth when Nature Extraction is a Function of Capital

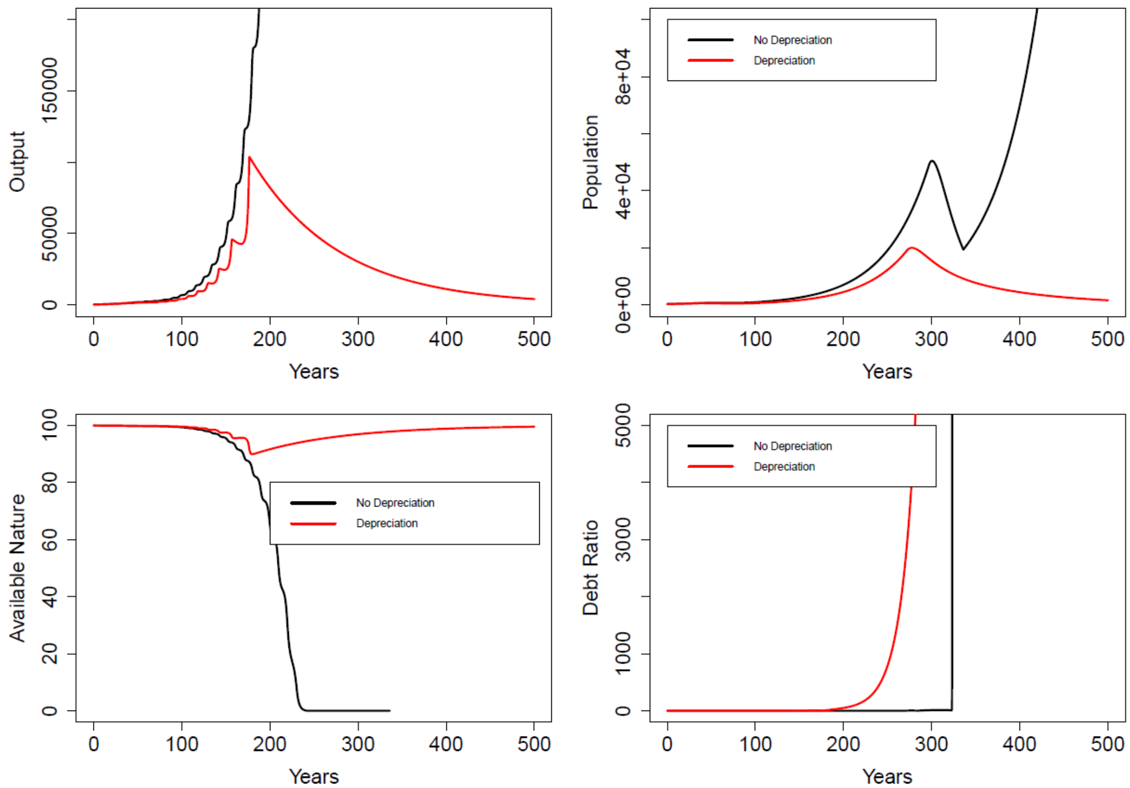

Figure 36. Simulating results for when nature is a function of capital with and without depreciation of wealth in the model. Here a nonlinear form of the Phillips curve is considered to model wages with a nonlinear form for investment including debt. The baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7} capital^{-1} time^{-1}$ is same as in section 5.2.

5.4.3. Comparing Results with and without Depreciation of Wealth when Nature Extraction is a Function of Power Input

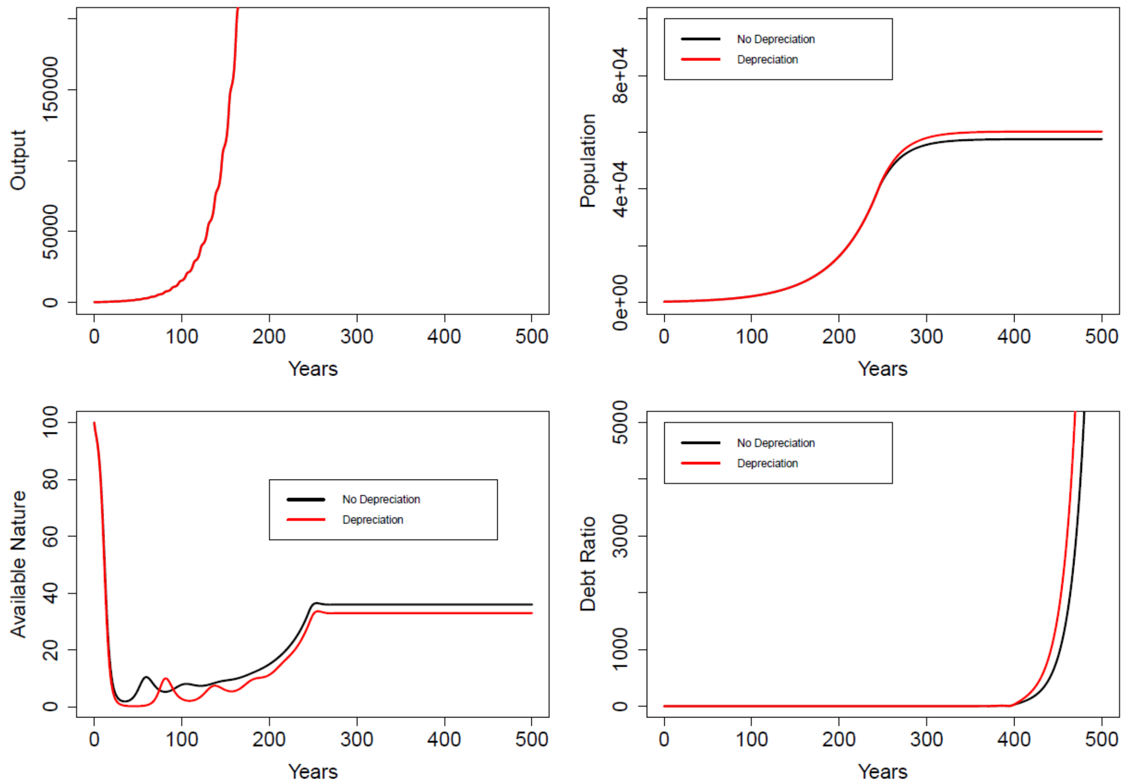

Figure 37. Simulating results for when nature is a function of power input with and without depreciation of wealth in the model. Here a nonlinear form of the Phillips curve is considered to model wages with a nonlinear form for investment including debt. The baseline extraction rate $\delta_{Pi,o} = 0.3335\ (power\ input)^{-1} time^{-1}$ is same as in section 5.3.

6. CHAPTER 6: DISCUSSIONS

6.1. OVERVIEW

In the previous sections, we have seen different scenarios for when nature extraction is a function of labor, capital, power input and the impact debt has in each case. In this section, I perform additional sensitivity in the model by increasing the constant rate of interest on debt from 5% to 15%. I will also compare the results with when there is no debt in the model and clearly highlight how critical the role of debt is.

6.1.1. EFFECT OF DEBT AT HIGH CONSTANT RATE OF INTERESTS WHEN NATURE EXTRACTION IS A FUNCTION OF LABOR

When we increase the rate of interest on debt from 5 to 15% the population begins to decline sooner at a higher value of interest rates (Figure 39). Declining population also results in an early regeneration of nature and therefore results in it returning back to its original value (Figure 38). However, the population and nature both seem to reach a steady state without debt in the system, clearly highlighting that a debt-induced collapse has occurred in the model.

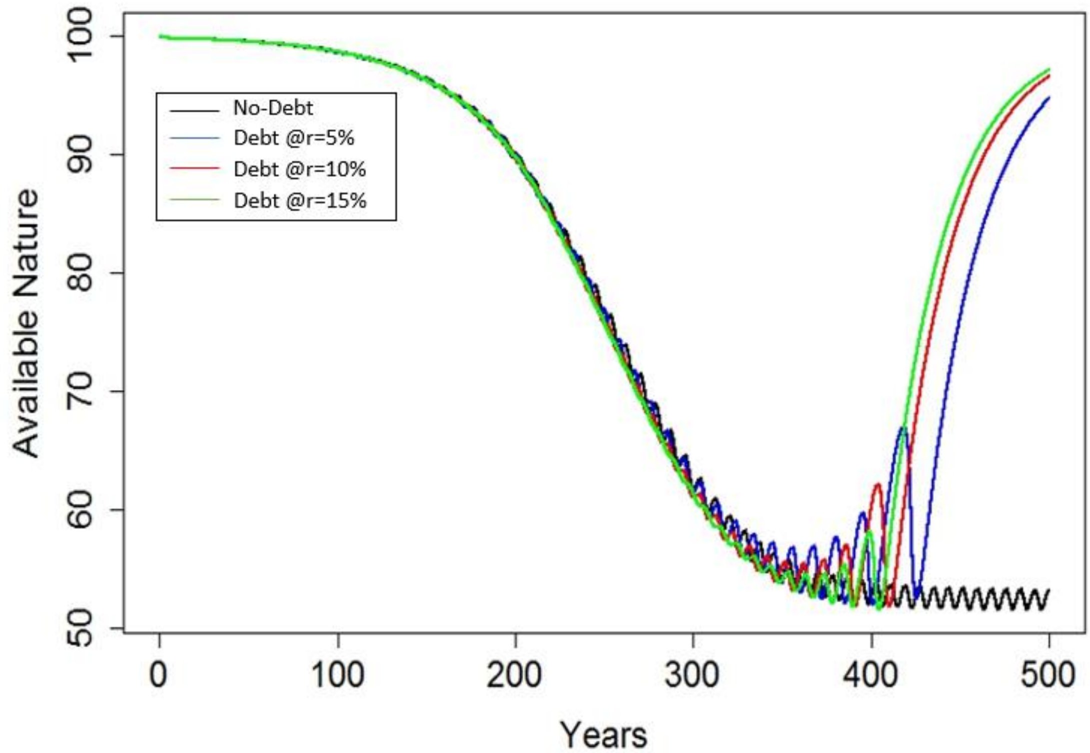

Figure 38. Simulating nature as a function of labor including debt for interest rates 5% to 15% and no debt at a rate of extraction of nature $\delta_{L,o} = 6.67\times10^{-6} person^{-1} time^{-1}$. Higher interest rates lead to an earlier collapse of nature while reaching a steady state without debt.

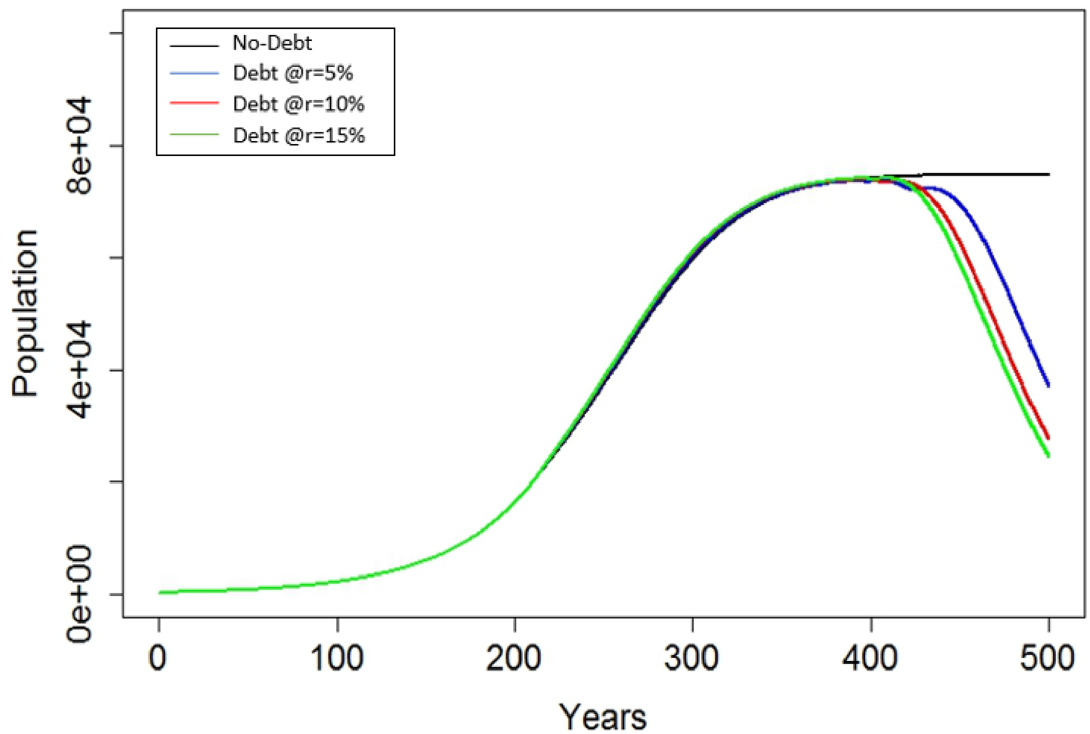

Figure 39.Simulating nature as a function of labor including debt for interest rates 5% to 15% and no debt at a rate of extraction of nature $\delta_{L,o} = 6.67 \times 10^{-6} person^{-1} time^{-1}$. Higher interest rates lead to an earlier collapse of population while reaching a steady state without debt.

6.1.2. Effect of Debt at high constant rate of interests when nature extraction is a function of Capital

In the case of assuming nature extraction is only a function of capital, when we increase the rate of interest on debt from 5 to 15%, the population declines sooner at high-interest rates (Figure 41), similar to that in the previous section 6.1.1. The difference here is that nature doesn't regenerate at low-interest rates and neither reaches a steady state when there is no debt in the model (Figure 40). The reasoning behind which has been discussed earlier in the sections 5.2.3 and 5.2.4.

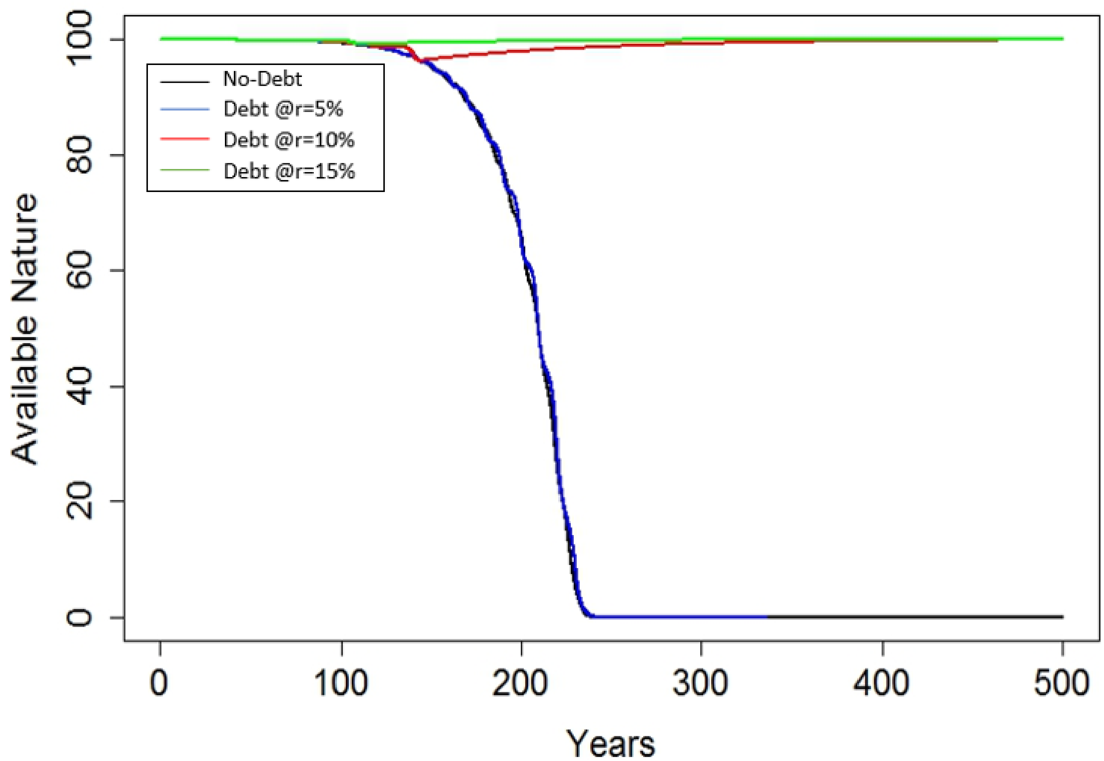

Figure 40. Simulating nature as a function of capital including debt for interest rates 5% to 15% and no debt at a rate of extraction of nature $\delta_{K,o} = 3.335\times 10^{-7} capital^{-1} time^{-1}$. Higher interest rates lead to an earlier collapse of nature whereas the nature is fully extracted at low interest rates and no debt.

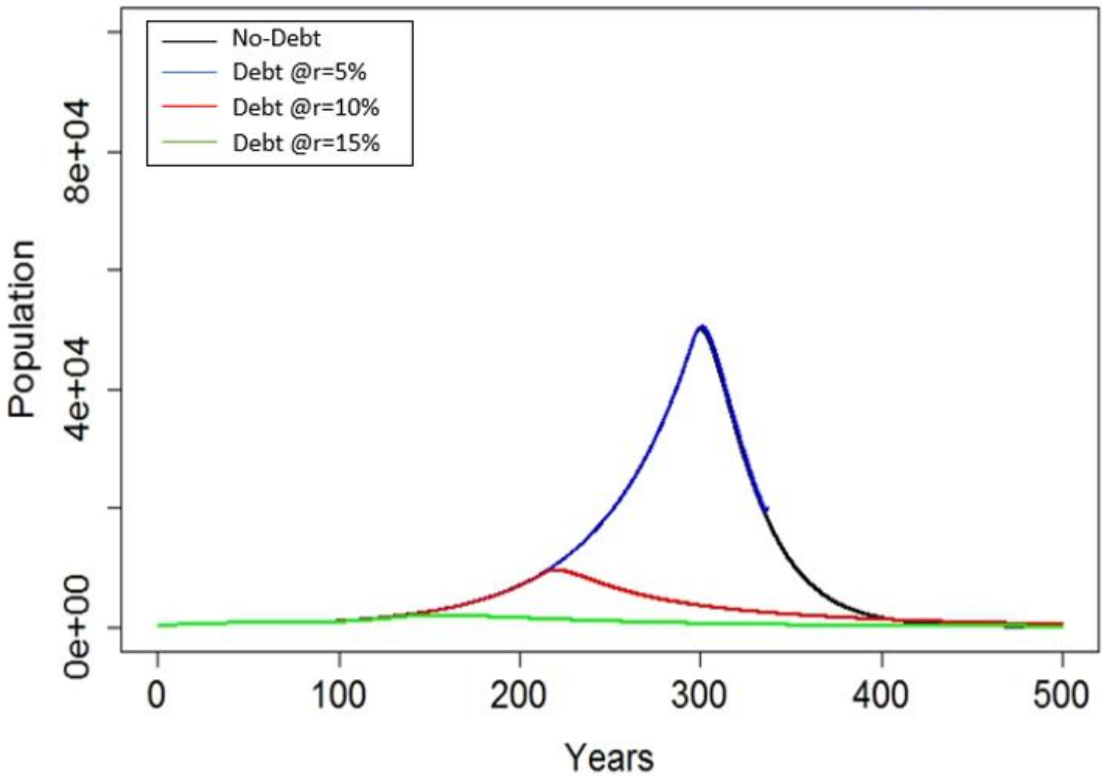

Figure 41. Simulating nature as a function of capital including debt for interest rates 5% to 15% and no debt at a rate of extraction of nature $\delta_{K,o} = 3.335\times 10^{-7} capital^{-1} time^{-1}$. Higher interest rates lead to an earlier collapse of population.

6.1.3. Effect of Debt at high constant rate of interests when nature extraction is a function of Power Input

In this section, we will discuss the impact of the high constant rate of interests on debt when nature extraction is a function of power input. When the constant rate of interest r on debt is increased from 5 to 15%, no effect is observed in the population (Figure 43) and similarly in the available nature (Figure 42). A very similar behavior was also observed when the rate of interest on the debt was varied from 1 to 5% (Figure A-16). The reasoning behind which has been discussed in section 5.3.2.

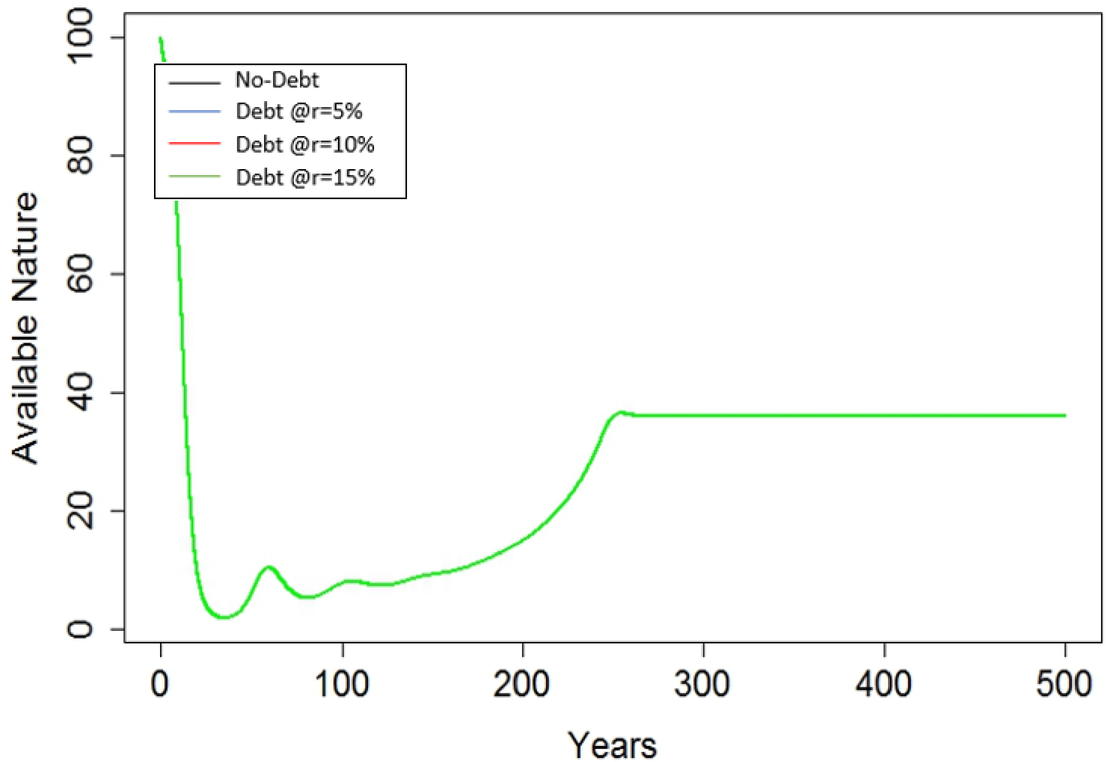

Figure 42. Simulating nature as a function of Power Input including debt for interest rates 5% to 15% and no debt at a rate of extraction of nature $\delta_{K,o} = 0.3335\, power\ input^{-1} time^{-1}$. Debt or No-Debt there is no affect in the model, available nature reaches steady state eventually. This is mainly because there is no feedback from the economic model to the parameters in the biophysical model when nature is a function of Power Input (Eq 68).

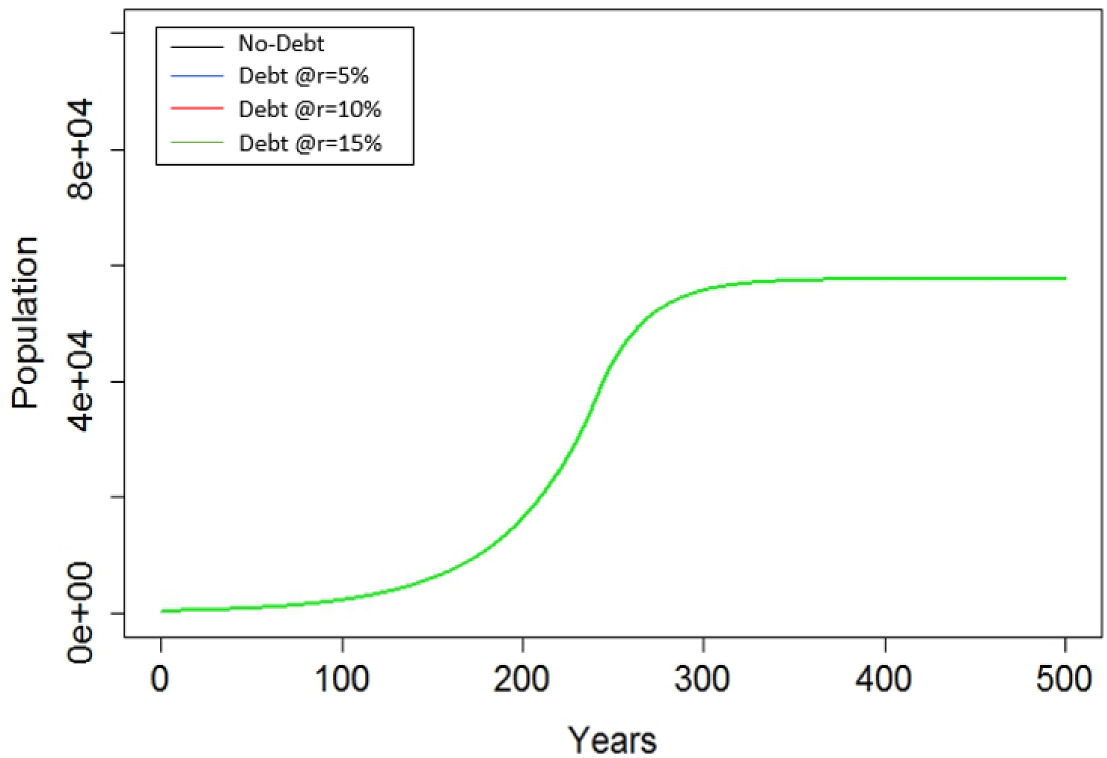

Figure 43. Simulating nature as a function of Power Input including debt for interest rates 5% to 15% and no debt at a rate of extraction of nature $\delta_{K,o} = 0.3335\ power\ input^{-1} time^{-1}$. Debt or No-Debt there is no affect in the model, the population reaches a steady state eventually. This is mainly because there is no feedback from the economic model to the parameters in the biophysical model when nature is a function of Power Input (Eq 68).

6.2. CONCLUSIONS

Debt has a direct impact in the merged biophysical and economic model when nature extraction is either a function of labor (section 5.1) or capital (section 5.2) but no impact when it's a function of power input (section 5.3). The model is also sensitive to other parameters like the constant rate of interest on debt, the linear and the nonlinear form of the Phillips curve and the rate of extraction of nature. There is a positive correlation between the constant rate of interest on debt and the speed of collapse occurring in the model. Higher values for interest rates lead to an earlier collapse of the model. There is also a correlation with the rate of extraction of nature where a collapse happens earlier at higher values and gets delayed at lower values.

I investigated the effect of assuming either a linear or a nonlinear form for the Phillips curve to model wages. It is important to notice that both seem to simulate very similar results (Figure 23 to Figure 26) with one possible exception: when we model nature as a function of capital including debt with the nonlinear form of the Phillips curve (section 5.2.4). The model no longer simulates realistic outcomes breaks ~330 years (Figure 30) (i.e. when wealth reaches zero but population explodes beyond which the results don't make any conceivable sense).

Another interesting observation (for the case when nature extraction is a function of capital including debt and the nonlinear form for the Phillips curve) is that for a given set of model parameters, there is a threshold interest rate above which nature cannot be fully depleted before a debt-induced collapse. For example, at 5% constant interest rate (Figure 30), nature is fully depleted before debt prevents profitability, but there is a debt-induced collapse at a higher constant interest rate of 7% (Figure 31) where employment and profits go to zero but nature is not fully depleted.

A much deeper analysis is required before I can conclude which form of the Phillips curve is best tailored for the model described in this thesis. suits the best to understand the different dynamics happening in the model.

6.3. LIMITATIONS

There are various limitations to our model at its current stage. A few of them are as listed below:

1. Our model doesn't replicate any real-world numbers for population, available nature (resources), output, labor, and etc. This model is at a very initial stage to depict and thus does not represent any real-world parameters.
2. The nonlinear form for the Phillips curve and the investment curve doesn't always keep the wage share and employment rate in bounds (i.e. 0% ≤ Employment Rate ≤ 100% and 0% ≤ Wage Share ≤ 100%) especially when nature extraction is a function of capital.
3. Our model assumes the assumptions inherited from both the individual models and is limited by their shortcomings.
4. The units of nature and wealth are both defined in terms of units of nature in the "HANDY" model and don't relate to real world monetary values.

6.4. FUTURE WORK

Considering the limitations in the previous section there is a significant amount of future work that can be performed on the model like:

1. The nature extraction function could be a combination of one or more of either labor, capital, and power input in the model.
2. A variable rate of interest can be introduced into the model where it would be a function of various different parameters in the model. This could help capitalists to

target a certain profit share by adjusting the different parameters that affect the interest rate in the model.

3. A bank/ financial system like that of a Godley table (Godley and Lavoie 2006) can be introduced to keep track of monetary flows in the model according to various accounting rules.

4. Debt can be further classified into different types like household debt, government debt, etc.

5. Age demographics can be included in the model to further classify the type of population that exists in the model.

7. APPENDIX A-FIGURES & TABLES

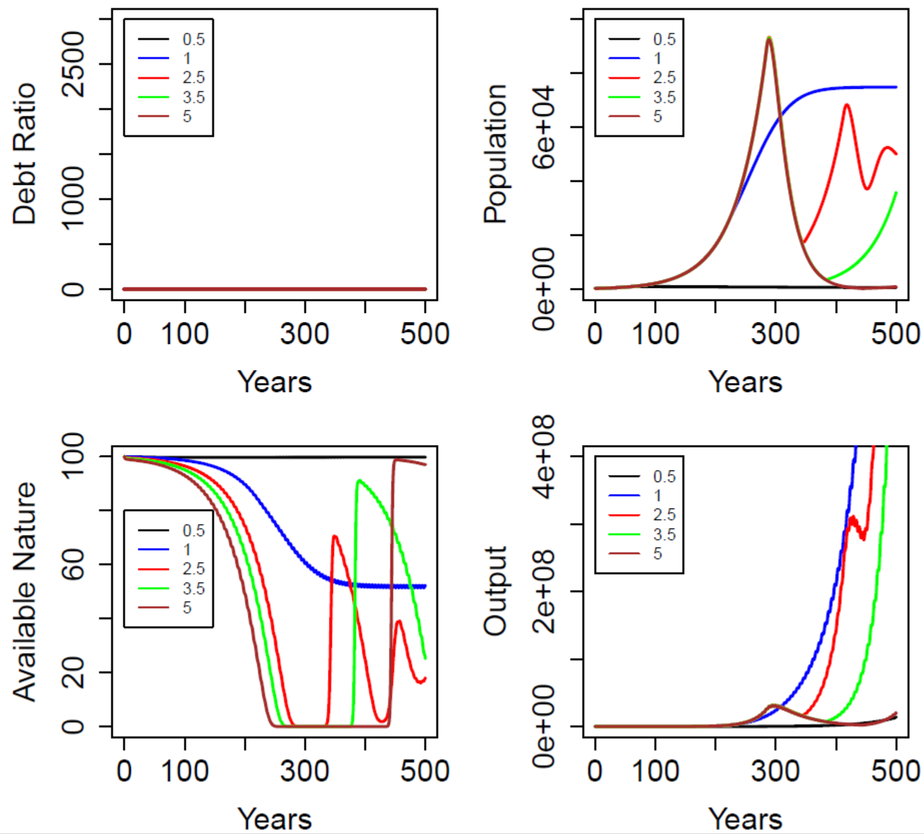

Figure A-1. Simulation results for when extraction rate of Nature is a function of Labor, with a baseline extraction rate $\delta_{L,o} = 6.67{\times}10^{-6}$ being varied from $0.5\delta_{L,o}\ to\ 5\delta_{L,o}$ without debt and wages being modeled according to a Linear form of the Phillips curve.

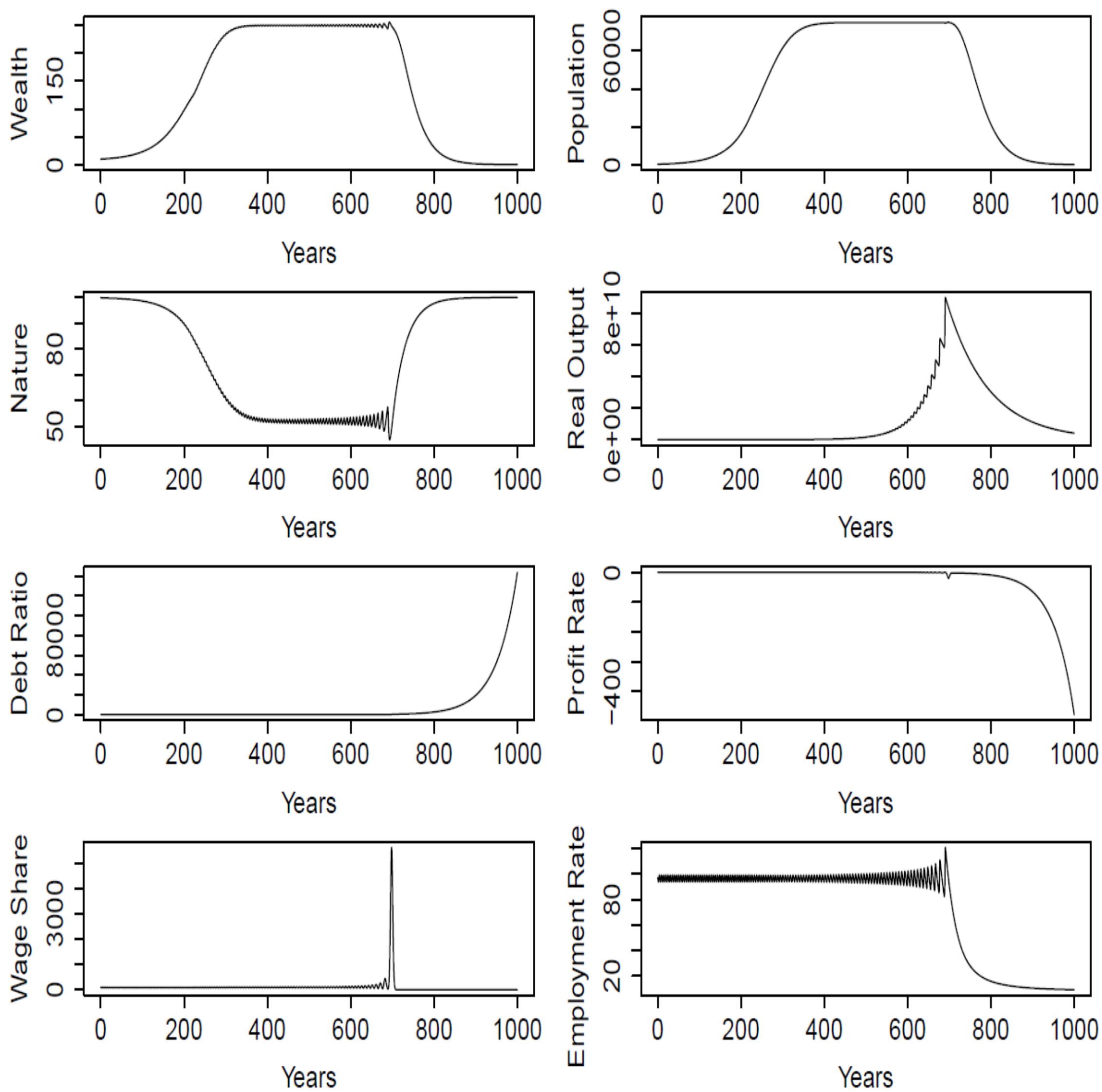

Figure A-2. Simulation results for 1000 years when extraction rate of nature is a function of Labor, with the baseline extraction rate $\delta_{L,o} = 6.67\times10^{-6} person^{-1} time^{-1}$ a linear Phillips curve, and debt accumulating at a constant interest rate of 1%/yr.

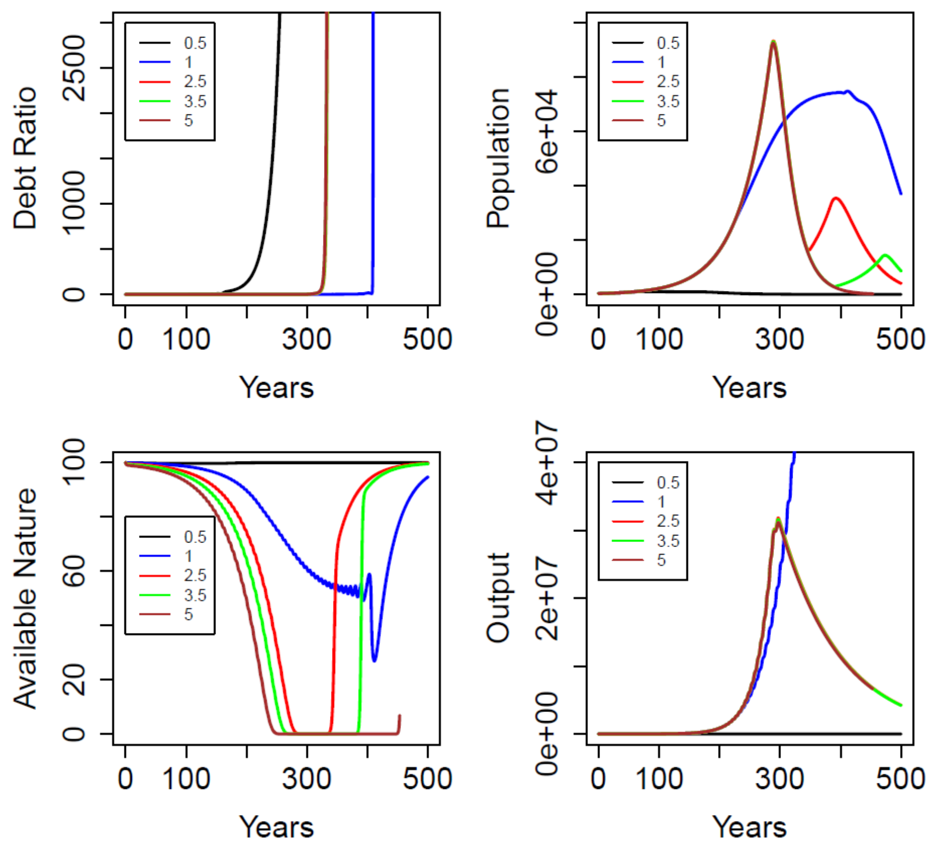

Figure A-3. Simulation results for when extraction rate of Nature is a function of Labor, with a baseline extraction rate $\delta_{L,o} = 6.67{\times}10^{-6}$ being varied from $0.5\delta_{L,o}\ to\ 5\delta_{L,o}$ with debt accumulating at a constat rate of interest of $r = 5\%$ and wages being modeled according to a linear form of the Phillips curve.

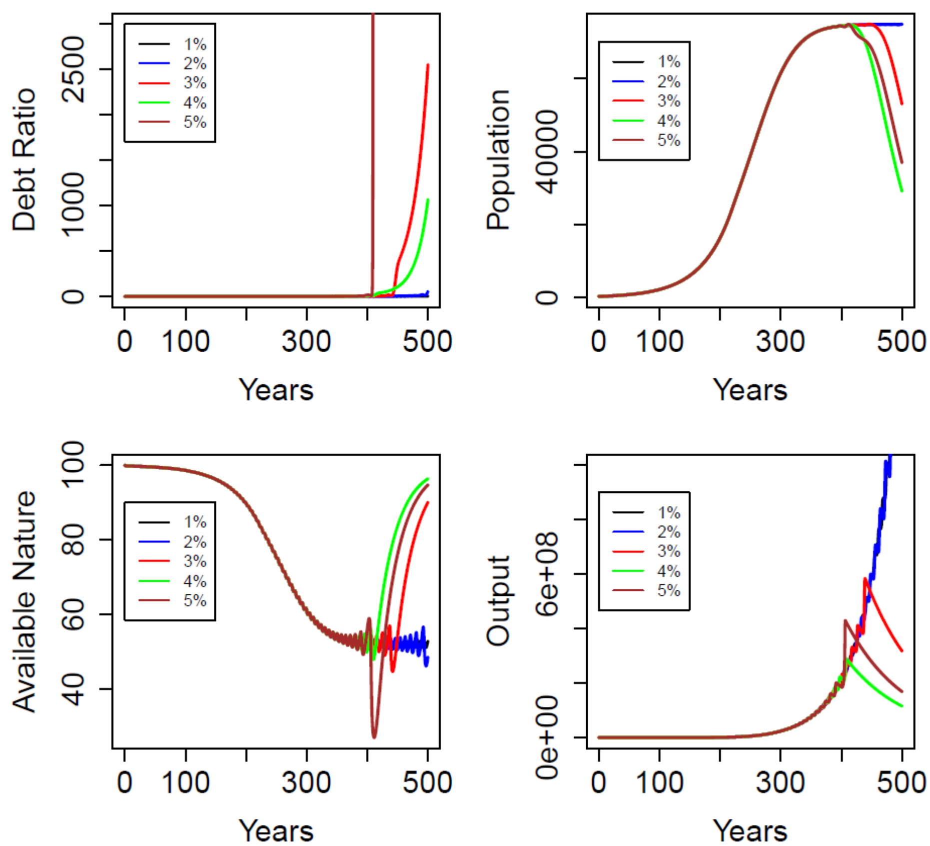

Figure A-4. Simulation results for when extraction rate of nature is a function of Labor, with a baseline extraction rate $\delta_{L,o} = 6.67\times10^{-6}$. While a constant rate of interest r on Debt is being varied from $1\%\ to\ 5\%$ with wages modeled according to the Linear form of Phillips curve.

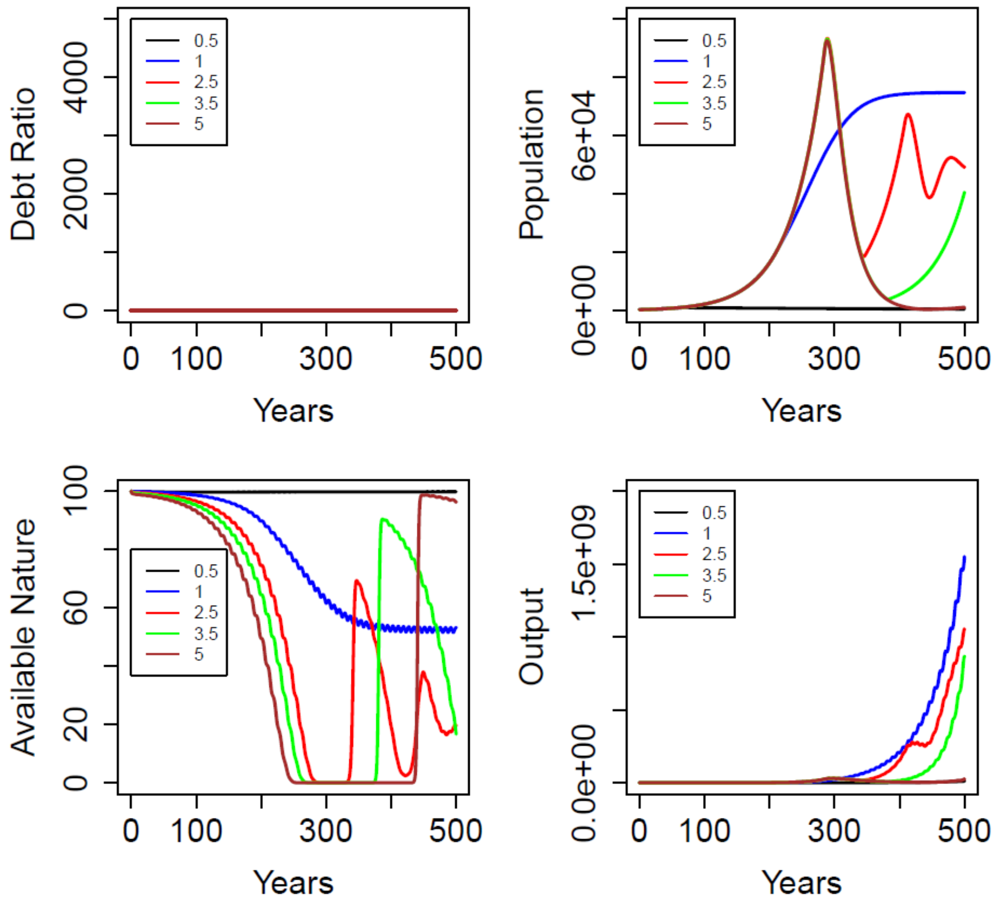

Figure A-5. Simulation results for when extraction rate of nature is a function of Labor, with the baseline extraction rate $\delta_{L,o} = 6.67{\times}10^{-6}$ being varied from $0.5\delta_{L,o}\ to\ 5\delta_{L,o}$ and wages being modeled according to a noninear form of the Phillips curve.

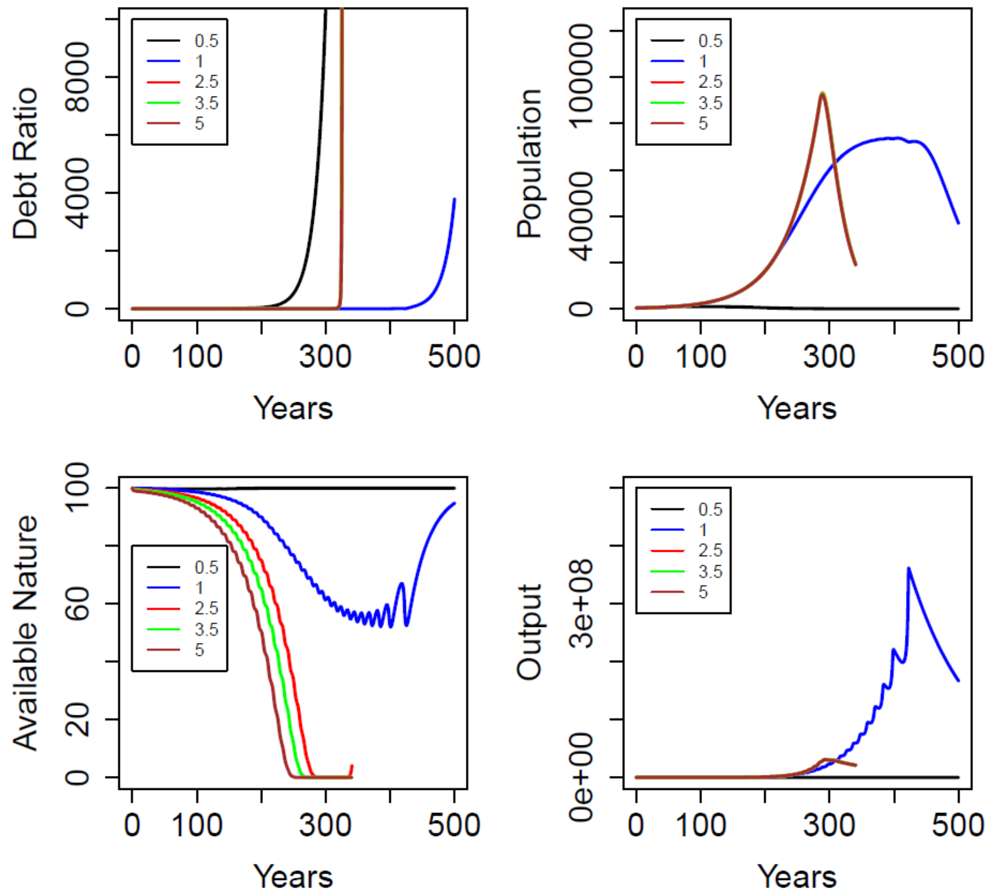

Figure A-6. Simulation results for when extraction rate of nature is a function of Labor, with a baseline extraction rate $\delta_{L,o} = 6.67\times10^{-6}$ being varied from $0.5\delta_{L,o}$ to $5\delta_{L,o}$ with debt accumulating at a constat interest rate, $r = 5\%$ and wages modeled according to a nonlinear form of the Phillips curve.

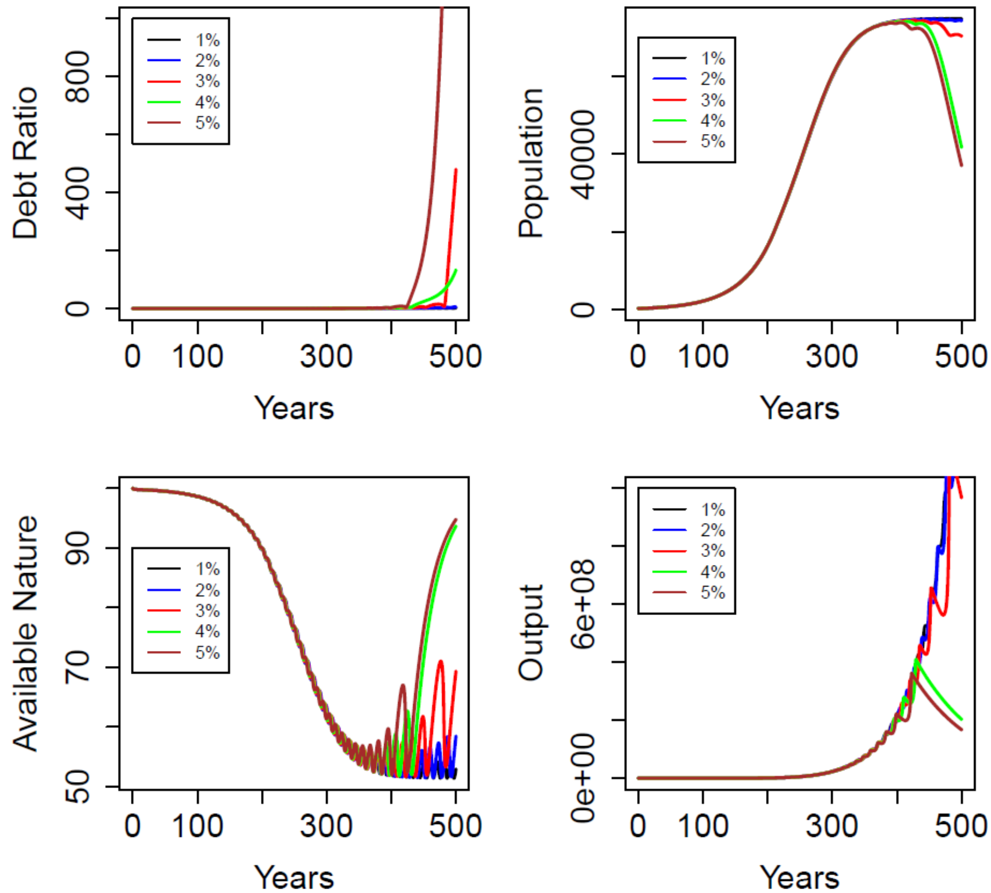

Figure A-7. Simulation results for when extraction rate of nature is a function of Labor, with the baseline extraction $\delta_{L,o} = 6.67\times10^{-6}$ including debt and the constat rate of interest r is being varied from $1\%\ to\ 5\%$ and wages being modeled according to a nonlinear form of the Phillips curve.

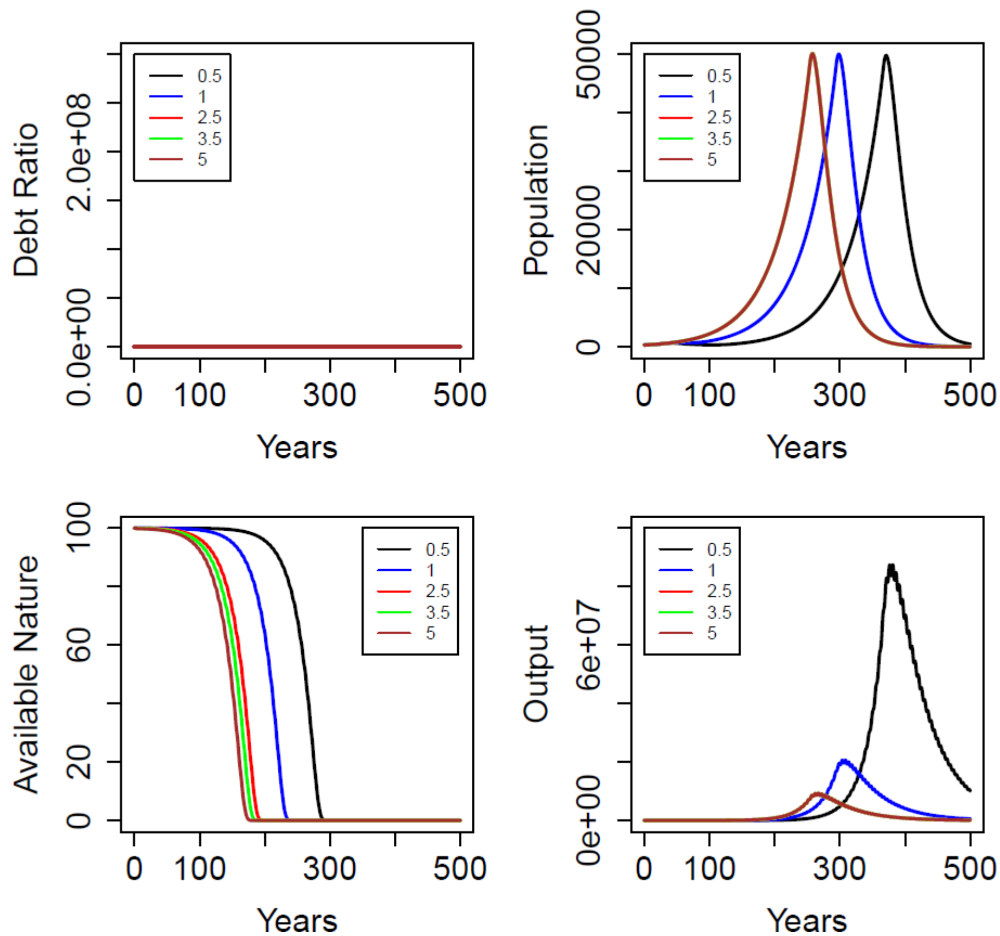

Figure A-8. Simulation results for when extraction rate of nature is a function of Capital, with the baseline extraction rate $\delta_{K,o} = 3.335 \times 10^{-7}$ being varied from $0.5\delta_{K,o}$ *to* $5\delta_{K,o}$ without debt and using a linear form of the Phillips curve to model wages.

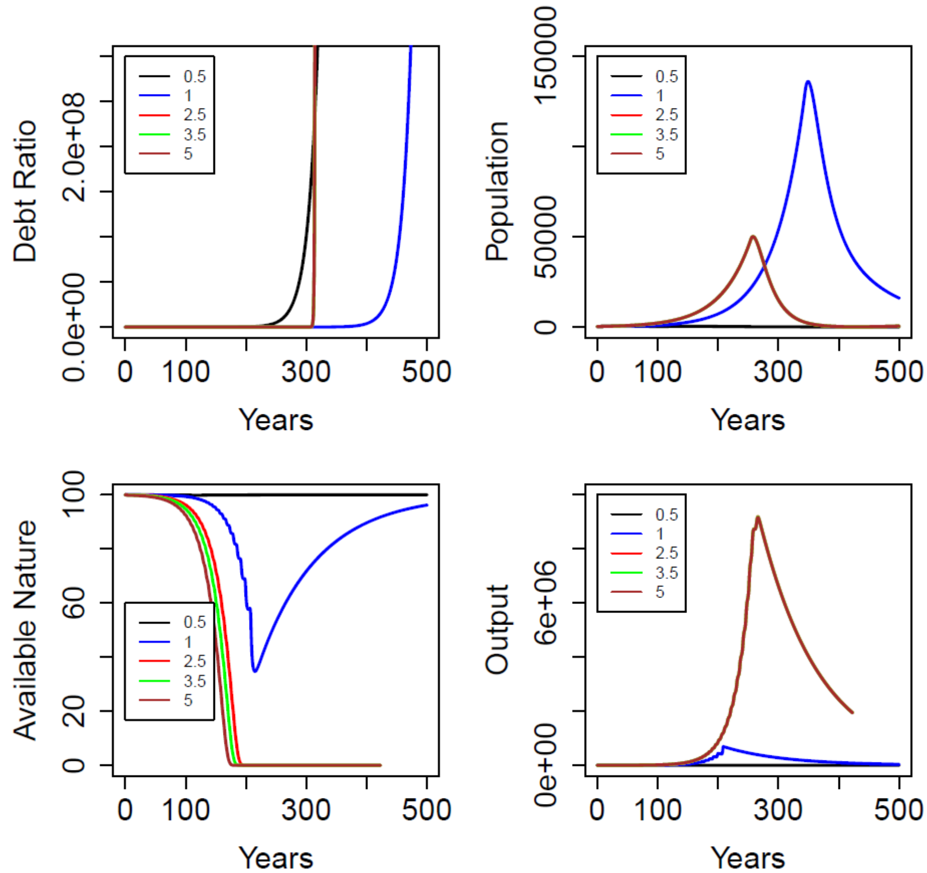

Figure A-9. Simulation results for when extraction rate of nature is a function of Capital, with the baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7}$ being varied from $0.5\delta_{K,o}\ to\ 5\delta_{K,o}$ including debt and a linear form of the Phillips curve to model wages.

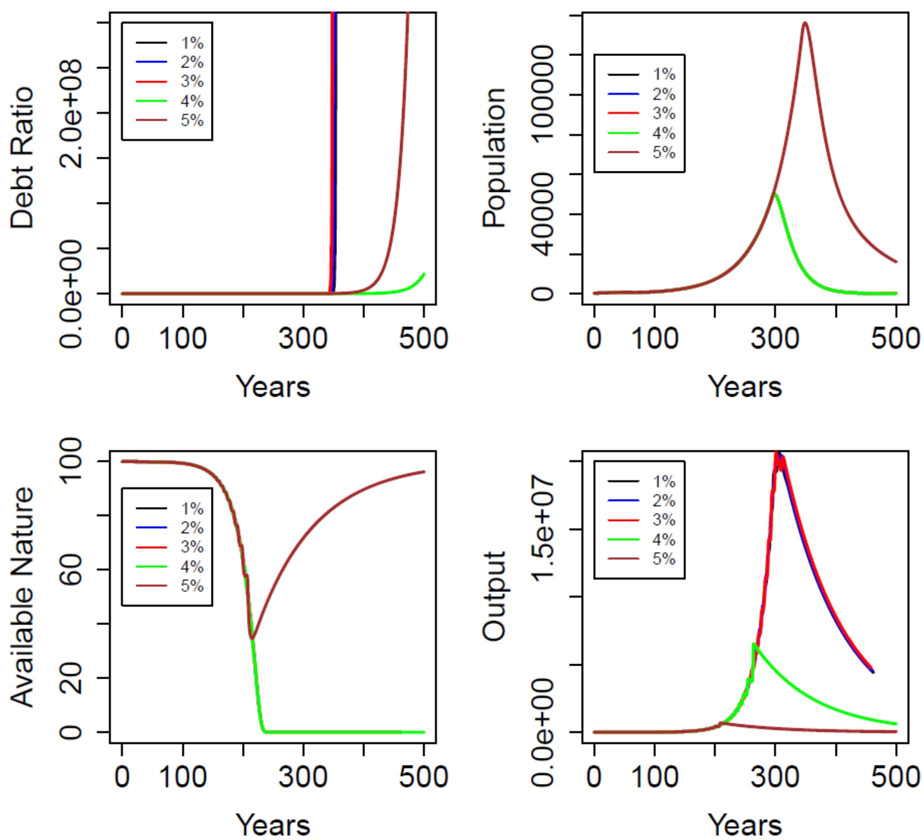

Figure A-10. Simulation results for when extraction rate of nature is a function of Capital, with the baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7}$including debt and the constant rate of interest, r is varied from $1\%\ to\ 5\%$ using a linear form of the Phillips curve to model wages.

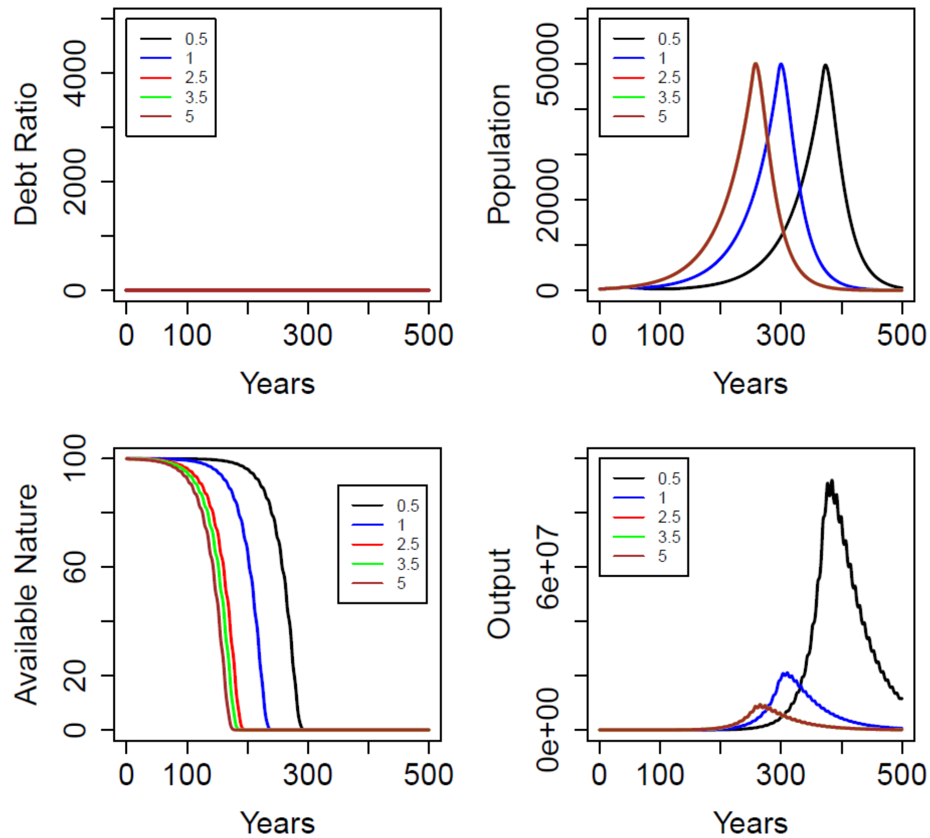

Figure A-11. Simulation results for when extraction rate of nature is a function of Capital, with the baseline extraction rate $\delta_{K,o} = 3.335 \times 10^{-7}$ being varied from $0.5\delta_{K,o}\ to\ 5\delta_{K,o}$ with no debt and a nonlinear form of the Phillips curve to model wages.

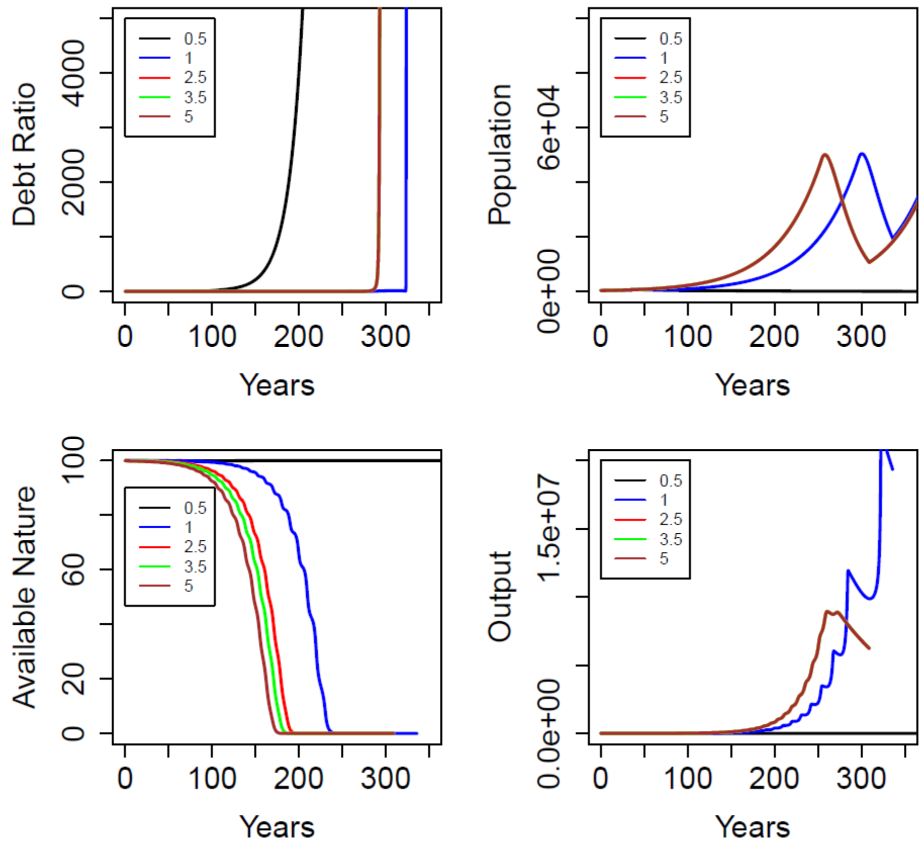

Figure A-12. Simulation results for when extraction rate of nature is a function of Capital, with a baseline extraction rate $\delta_{K,o} = 3.335\times10^{-7}$ being varied from $0.5\delta_{K,o}\ to\ 5\delta_{K,o}$ including debt and a nonlinear form of the Phillips curve to model wages.

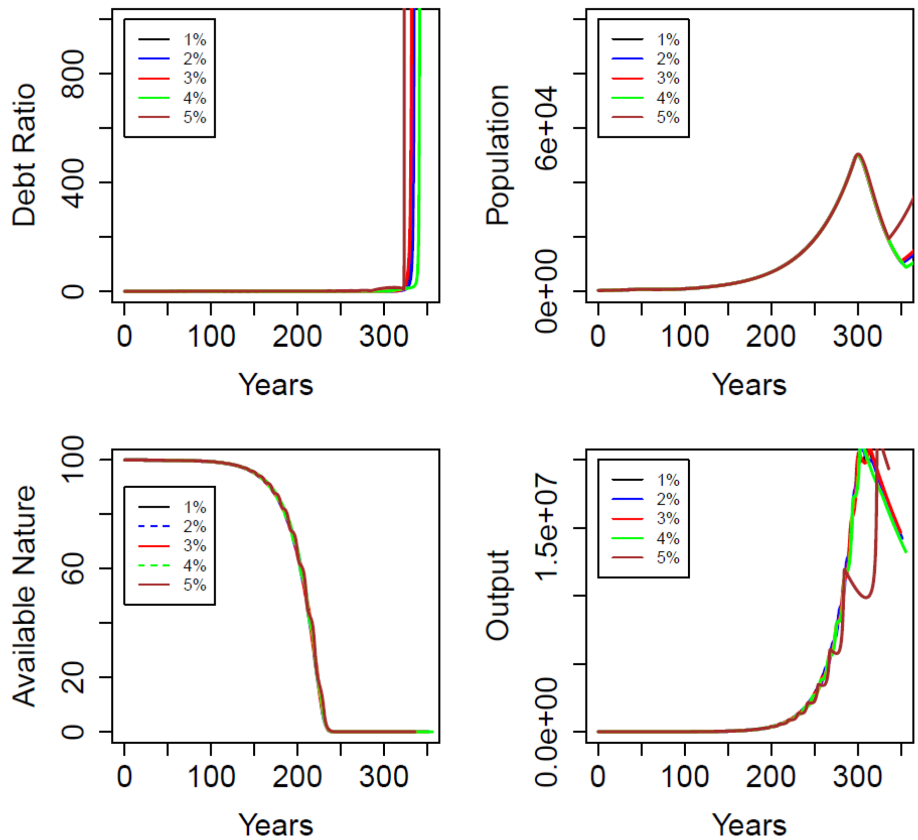

Figure A-13. Simulation results for when extraction rate of nature is a function of Capital, with the baseline extraction rate $\delta_{K,o} = 3.335 \times 10^{-7}$ including debt and the constant rate of interest r is being varied from $1\%\ to\ 5\%$ using a nonlinear form of the Phillips curve to model wages.

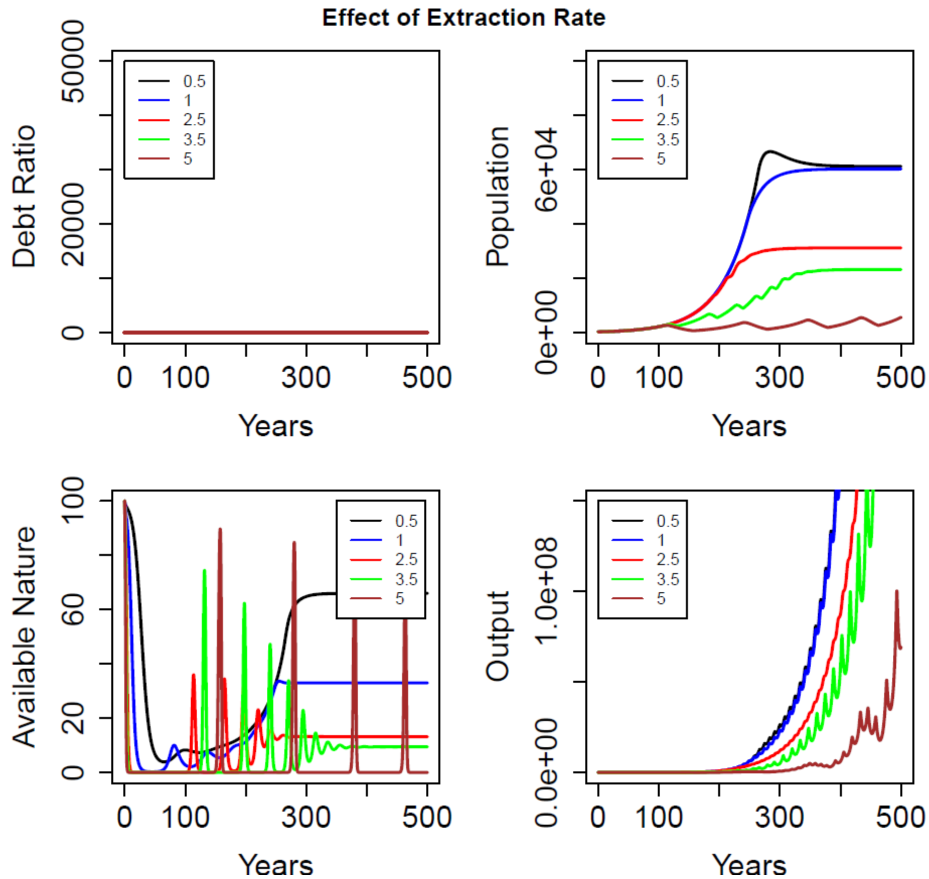

Figure A-14. Simulation results for when extraction rate of nature is a function of Power Input, with the baseline extraction rate $\delta_{Pi,o} = 0.3335\ (power\ input)^{-1}time^{-1}$ being varied from $0.5\delta_{Pi,o}\ to\ 5\delta_{Pi,o}$ without debt and a nonlinear form of the Phillips curve to model wages.

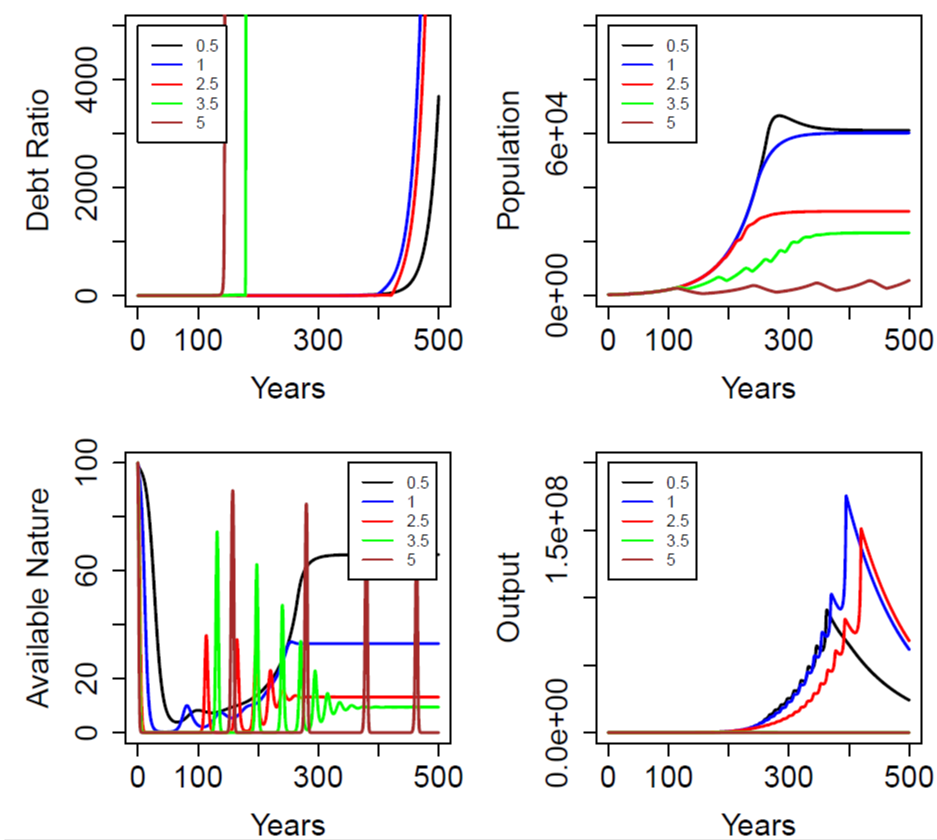

Figure A-15. Simulation results for when extraction rate of nature is a function of Power Input, with the baseline extraction rate $\delta_{Pi,o} = 0.3335$ being varied from $0.5\delta_{Pi,o}\ to\ 5\delta_{Pi,o}$ with debt being accumulated at a constant rate of interest, $r = 5\%$ and a nonlinear form of the Phillips curve to model wages.

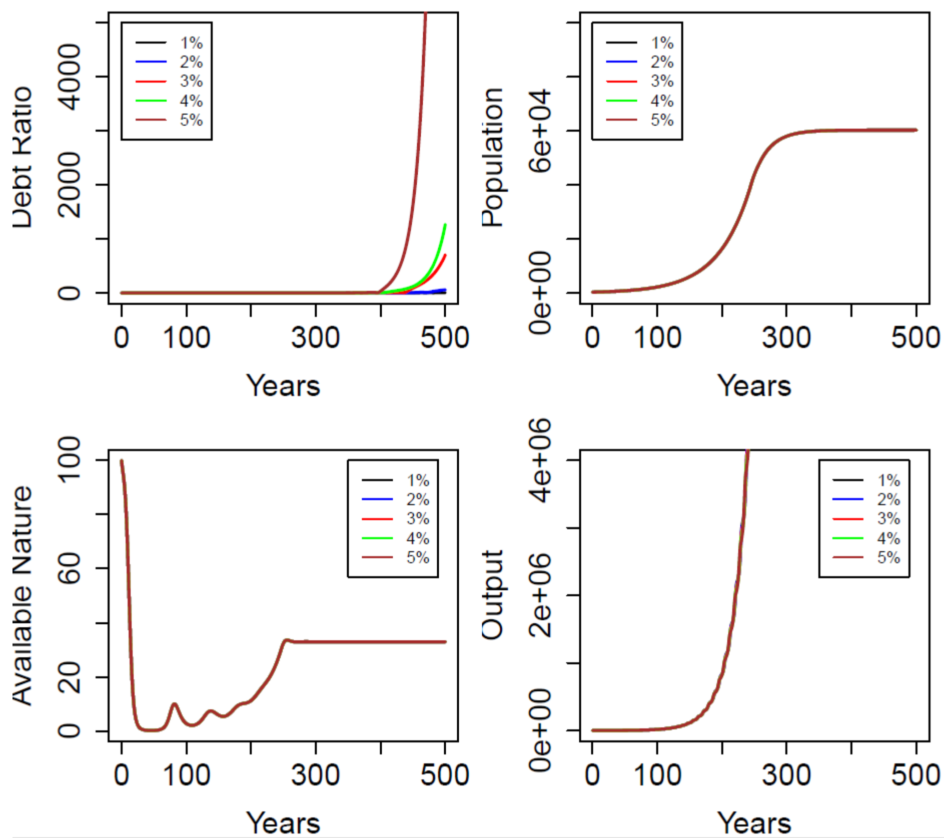

Figure A-16. Simulation results for when extraction rate of nature is a function of Power Input, with the baseline extraction rate $\delta_{Pi,o} = 0.3335\ (power\ input)^{-1} time^{-1}$ including debt and the constant rate of interest, r is varied from $1\%\ to\ 5\%$ with a nonlinear form of the Phillips curve to model wages.

Table A-1. Different parameters and variables with their values used to simulate the Goodwin model with or without debt including either a linear or a nonlinear form for the Phillips and the investment curve as in Keen, 2013.

Parameter Name	Parameter Symbol	Parameter units	Initial Values in the Model
Population	N	people	100
Labor	L	people	97
Labor productivity	a	\$/people	1
Output	Y	\$	97
Capital	K	\$	291
Debt	D	\$	0
Real Wage	w	\$/people	0.88
Rate of Growth of Labor Productivity	α	\$/people/time	0.02
Constant Rate of Interest	r	%/year	5
Depreciation of Capital	γ	/year	0.01
Capital to output ratio	v	-	3
Parameters for Linear Phillips curve			
Variable	**Description**	**Value**	
c	Constant in the Linear Phillips curve	4.8	
d	Constant in the Linear Phillips curve	5	
Parameters for Nonlinear Phillips curve			
Parameter	**Description**		

$P_h(\lambda) = GenExp(\lambda, 0.95, 0.0, 0.5, -0.01)$	Different parameters for the nonlinear Phillips curve in the Minsky model
$I(\pi_r) = GenExp(\pi_r, 0.05, 0.05, 1.75, 0)$	Different parameters for the nonlinear investment function in the Minsky model

Table A-2. Different parameters and variables with their values used to simulate the "HANDY" model as in (Motesharrei et. al, 2014).

Parameter Name	Parameter Symbol	Parameter units	Initial Values in the Model
Commoner Population	x_C	people	100
Elite Population	x_E	people	0
Birth Rate of Commoners	β_C	/year	0.03
Birth Rate of Elites	β_E	/year	0.03
Normal (Minimum) Death Rate	α_m	/year	0.01
Famine (Maximum) Death Rate	α_M	/year	0.07
Inequality Factor	κ	-	0,1,10,100
Regeneration rate of Nature	γ	/Eco-$/year	0.01
Nature Carrying Capacity	λ	Eco-$	$1\times10^{+2}$
Nature	y	Eco-$	λ
Regeneration Rate of Nature	γ	/Eco-$/year	0.01
Depletion(Production) factor	δ	/people/year	6.67×10^{-6}
Accumulated wealth	w	Eco-$	0
Threshold wealth per Capita	ρ	Eco-$/people	5×10^{-3}
Subsistence Salary per Capita	s	Eco-$/people/year	5×10^{-4}

Table A-3. Different parameters and variables with their initial values used to simulate the merged model ("HANDY" + "Goodwin + Debt") as in (Motesharrei et. al, 2014 and Keen, 2013).

Parameter Name	**Parameter Symbol**	**Parameter units**	**Initial Values in the Model**
Total Population	x_{hc}	people	300
Birth Rate	β_{hc}	/year	0.03
Normal (Minimum) Death Rate	α_m	/year	0.01
Famine (Maximum) Death Rate	α_M	/year	0.07
Subsistence Salary per Capita	s	Eco-$/people/year	5×10^{-4}
Regeneration rate of Nature	γ_{hc}	/Eco-$/year	0.01
Nature Carrying Capacity	λ_h	Eco-$	$1\times10^{+2}$
Nature	y_h	Eco-$	λ_h
Depletion (Production) factor	$\delta_{K,o}$	/people/year	$1\times10^{+2}$
Depletion (Production) factor	$\delta_{L,o}$	/capital/year	6.67×10^{-6}
Depletion (Production) factor	$\delta_{Pi,o}$	/power input/year	0.3335
Accumulated wealth	w_h	Eco-$	10
Depreciation of Wealth	δ_h	/year	0.01
Threshold wealth per Capita	ρ	Eco-$/people	5×10^{-3}
Output	Y	$	291
Labor	L	people	291
Labor productivity	a	$/people	1
Depreciation of Capital	γ	/year	0.01

Rate of Growth of Labor productivity	α	\$/people	0.02
Real Wage	w	\$/people	0.95
Capital	K	\$	873
Debt	D	\$	0
Constant Rate of Interest	r	%/year	1, to 5, 10, 15, 20
Power Factor	pf	%/year	1, 5, 10, 15, 20
Capital to output ratio	v	-	3

Parameters for Linear Phillips curve

Variable	Description	Value
c	Constant in the Linear Phillips curve	4.8
d	Constant in the Linear Phillips curve	5

Parameters for Nonlinear Phillips curve

Parameter	Description
$P_h(\lambda) = GenExp(\lambda, 0.95,0.0,1.6, -0.05)$	Different parameters for the nonlinear Phillips curve in the Minsky model
$I(\pi_r) = GenExp(\pi_r, 0.03333,0.1,2.25,0)$	Different parameters for the nonlinear investment function in the Minsky model

8. APPENDIX B -R CODES

8.1. CODES TO SIMULATE THE HUMAN AND NATURE DYNAMICS MODEL:

```
##This code runs the "HANDY" model as in Motesharrei et al., 2014
library(deSolve)
handy <- function(t,y,parms){
  ##The Four Variables defined as States in the
Model################################
  x_c <- y[1];  # Population of Commoners
  x_e <- y[2];  # Population of Elites
  y_h <- y[3];  # Available Nature
  w <- y[4];    # Wealth accumulated

#####################################################################
################
  w_th <- rho*x_c + kappa*rho*x_e; # Threshold wealth as defined in the paper
  w_z <- w/w_th; # Variable defined as wealth over the threshold wealth in the
model
  C.c <- (pmin(1,w_z))*S*x_c;# Consumption by commoners
  C.e <- (pmin(1,w_z))*kappa*S*x_e;# Consumption by elites
  ##Variables defined to avoid confusion with many
equations######################
  v.c <- 1-(C.c/(S*x_c));
  v.e <- 1-(C.e/(S*x_e));

#####################################################################
##############
```

```
  # Death rate of commoners and elites incluing the concept of
famine############
  alpha_c <- alpha.m + ((pmax(0,v.c,na.rm = TRUE))*
                  (alpha.M - alpha.m));
  alpha_e <- alpha.m + ((pmax(0,v.e,na.rm = TRUE))*
                  (alpha.M - alpha.m));

######################################################################
################
##State equations as in the paper that determine the simulation
  xc_dot <- beta.c*x_c - alpha_c*x_c;
  xe_dot <- beta.e*x_e - alpha_e*x_e;
  yh_dot <- gamma*y_h*(lambda - y_h) - delta*x_c*y_h;
  w_dot <- delta*x_c*y_h - C.c - C.e;
  list(c(xc_dot,xe_dot,yh_dot,w_dot))
}
######################################################################
#######################
assign("alpha.m",0.01);        # [1/yr] normal (minimimum) death rate
assign("alpha.M",0.07);        # [1/yr] Famine (maximum) death rate
assign("beta.c",0.03);         # [1/yr] commoner birth rate
assign("beta.e",0.03);         # [1/yr] elite birth rate
assign("S",5e-4);              # [nature/(people * yr)] subsitence salary per people
assign("rho",5e-3);            # [nature/people] subsitence salary per capita
assign("gamma",0.01);          # [1/(nature * yr)] regeneration rate of nature
assign("lambda",100);          # [nature] nature carrying capacity
assign("kappa",1);             # Inequality factor (multiple of elite consumption per
assign("delta",6.67e-6);       # [1/(commoner * yr)] depletion (production) factor
```

```
######################################################################
######################
xc_o <- 100;
xe_o <- 0;
yh_o <- lambda;
w_o <- 0;
######################################################################
######################
yini = c(xc_dot =xc_o,xe_dot = xe_o,yh_dot = yh_o,w_dot = w_o)
times <- seq(from = 0, to = 1000, by = 0.01)
handy_model <- ode(times = times,y = yini,
                func = handy, method = c("ode45"), parms = NULL)
# Outputs from the function to be plotted
time <- handy_model[,1];
commoner_population <- handy_model[,2];
elite_population <- handy_model[,3];
nature <- handy_model[,4];
wealth <- handy_model[,5];
# Plotting equations
par(mfrow = c(2,2))#mar = c(2,2,2,2))
bmar <- 4 # Bottom margin (1)
lmar <- 5 # Left margin (2)
tmar <- 1 # Top margin (3)
rmar <- 1 # Right margin (4)
plot(time,commoner_population, type = "l",col = "blue",
    xlab = "Time",ylab = "commoner_population",cex.lab=1.5,cex.axis=1.5 )
plot(time,elite_population, type = "l",col = "red",
    xlab = "Time",ylab = "elite_population",cex.lab=1.5,cex.axis=1.5 )
```

```
plot(time,nature, type = "l",col = "green",
     xlab = "Time",ylab = "nature",cex.lab=1.5,cex.axis=1.5 )
plot(time,wealth, type = "l",col = "black",
xlab = "Time",ylab = "wealth",cex.lab=1.5,cex.axis=1.5 )
```

8.2. CODES TO SIMULATE THE GOODWIN MODEL WITH DEBT AND WAGES BEING MODELED EITHER ACCORDING TO THE LINEAR OR THE NONLINEAR FORM OF THE PHILLIPS CURVE:

```
# This code simulates "The Goodwin Model" as in Keen, 2013 with and Without
Debt
# it also simulates its different forms i.e. the linear and the nonlinear form for the
Philip's Curve
#####################################################################
################################
## 1. type.wages = 1 simulates the Linear form for the Philip's Curve.
## 2. type.wages = 2 simulates the nonlinear form for the Philip's Curve.
## 3. type.debt = 1 simulates debt determined by the nonlinear investment curve
## 4. type.debt = 2 simulates no debt with linear relationship for Investments
#####################Where all profits are invested. Therefore, Invetments =
Profit
#####################################################################
#################################
library(deSolve)
goodwin <- function(t,y,parms){
#Various states defined in the goodwin model in Keen, 2013
  a <- y[1];# Labor Productivity
  Y <- y[2];# Output
  N <- y[3];# Population
```

```
  w <- y[4];# Real Wages
  D <- y[5];# Debt
######################################################################
################################
  L <- Y/a;#labor
  employment_rate <- L/N; # Employment Ratio (Lambda)
  Wages <- (w)*(L);       # Total Wages (Real Wages * Labor)
  profit <- Y-Wages-r*D;  # Profit
  profit_rate <- (profit/(v*Y));# Profit Rate
######################################################################
################################
  if (profit >= 0) {
    Invest_func <- (inv_yval - inv_min)*exp((inv_s/(inv_yval - inv_min))*
                                  (profit_rate-inv_xval)) + inv_min;
  } else {
    Invest_func <- 0
  }
######################################################################
#######################################
  if (type.wages==1) {
    ph_func <- (-c+d*employment_rate);#Linear Philip's Curve
  } else if (type.wages==2) {
    ph_func <- (ph_yval - ph_min)*exp((ph_s/(ph_yval - ph_min)) *
                               (employment_rate - ph_xval)) + ph_min;# Nonlinear
Philip's Curve
  }
######################################################################
#######################################
```

```
#Equations that determine the simulations in the
model#####################################################
  a_dot <- alpha*a;                        # Labor Productivity
  Y_dot <- ((Invest_func/v)-gamma_deprec)*Y;# Output
  N_dot <- beta*N;                         # Population
  w_dot <- ph_func*w;                      # Real Wages
###################################################################
################################
  if (type.debt==1) {
    Investment <- Invest_func*Y;# Nonlinear Investment Curve
  } else if (type.debt==2) {
    Investment <- profit;       # Linear Investment, where All profits are
Invested###############
  }
  D_dot <- Investment - profit; # Debt
  list(c(a_dot,Y_dot,N_dot,w_dot,D_dot));
}
###################################################################
############
assign("type.wages",2) # Linear or Nonlinear Philip's Curve
assign("type.debt",1)  # Debt or No-Debt
assign("c",4.8);       # Phlips Cuve = -c+d*Lambda
assign("d",5);
assign("v",3);         # Accelerator
assign("alpha",0.02);  # Labor Productivity Growth Rate
assign("beta",0.01);   # Population Growt Rate
assign("gamma_deprec",0.01); # Depreciation Rate of Capital
assign("r",0.05);      # Constant rate of Interest on Debt
```

```
##################################################################
#############
##Values for different Constants in the Nonlinear Philip's and Investment Curve
assign("ph_xval",0.95);
assign("ph_yval",0.0);
assign("ph_s",1.6);
assign("ph_min",-0.05);
assign("inv_xval",0.03333);
assign("inv_yval",0.1);
assign("inv_s",1*2.25);
assign("inv_min",0.0);
##################################################################
###############
###Setting initial conditions
a_o <- 1;       # Initial Labor Productivity
N_o <- 100;     # Intial  Population
L_o <- 0.97*N_o;# Initial Labor
Y_o <- a_o*L_o; # Initial Output
D_o <- 0;       # Initial Debt
w_o <- 0.88;    # Initial Real Wage
##################################################################
#####################
yini <- c(a=a_o,Y=Y_o,N=N_o,w=w_o,D=D_o);
times <- seq(from = 0, to = 500, by = 0.01);
out <- ode(times = times, y = yini, func = goodwin, method = c("ode45"), parms
= NULL);
#output for the different states in the
model##########################################
```

```
time <- out[,1];
labor_productivity <- out[,2];
real_output <- out[,3];
population <- out[,4];
real_wages <- out[,5];
Debt <- out[,6];
##################################################################
#####################################
capital <- real_output*v;
labor <- real_output/labor_productivity;
profit <- real_output-(real_wages*labor)-r*Debt;
profit_rate <- (profit/(v*real_output));
employment_rate <- (labor/population);
wage_share <- 100*(real_wages/labor_productivity);
Debt_ratio <- Debt/real_output;
ph_func <- (ph_yval - ph_min)*exp((ph_s/(ph_yval - ph_min)) *
(employment_rate - ph_xval)) + ph_min;
Invest_func <- (inv_yval - inv_min)*exp((inv_s/(inv_yval -
inv_min))*(profit_rate-inv_xval)) + inv_min;
##################################################################
#####################################
#Plotting Different States in the
model#############################################################
###
par(mfrow = c(2,2))
bmar <- 4 # Bottom margin (1)
lmar <- 5 # Left margin (2)
tmar <- 2 # Top margin (3)
```

```
rmar <- 1 # Right margin (4)
par(mar=c(bmar,lmar,tmar,rmar))
plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
plot(time,labor,xlab = "Years",ylab = "Labor",type="l",cex.lab=1.5,cex.axis=1.5)
plot(time,population,xlab = "Years",ylab =
"Population",type="l",col="black",cex.lab=1.5,cex.axis=1.5)
plot(100*employment_rate,wage_share,xlab = "Employment Rate %",ylab =
"Wages share of output %",type = "l",col="black",cex.lab=1.5,cex.axis=1.5)
```

8.3. Codes to Simulate the Merged Model ("HANDY" + "The Goodwin Model") with and without Debt, wages being modeled either according to the linear or the nonlinear form of the Phillips curve, and a nonlinear investment curve:

```
#This code is the master code for my thesis which has the ability to run the below
models
######################################################################
######################
#1. Handy merged with Goodwin model with linear Phillips curve with either
Debt or No Debt
###1.1 Nature extraction as a function of Labor(L)
###1.2 NAture extraction as a function of Capital(K)
#2. Handy merged with Goodwin model with general exp form for Phillips
curve(Keen,2013) with either Debt or No Debt
###2.1 Nature extraction as a function of Labor(L)
###2.2 NAture extraction as a function of Capital(K)
###2.3 NAture extraction as a function of Power Input(pi)
```

```
#3. Handy merged with Goodwin model with general exp form for Phillips curve,
Debt/No debt with depreciation of Wealth
###3.1 Nature extraction as a function of Labor(L)
###3.2 NAture extraction as a function of Capital(K)
###3.3 NAture extraction as a function of Power Input(pi)
library(deSolve)
goodwinhandy <- function(t,y,parms){
  #State Variables
  a <- y[1];   # Labor Productivity
  w_h <- y[2]; # Wealth
  Y <- y[3];   # Output
  x_hc <- y[4];# Total Population
  w <- y[5];   # Real wages
  y_h <- y[6]; # Available Nature
  D <- y[7];   # Debt
  L <- Y/a;    # Output
  K <- Y*v;    # Capital

#######################################################################
#################
  w_th <- rho*x_hc; # Threshold wealth as defined in the paper
  w_z <- w_h/w_th;  # Variable defined as wealth over the threshold wealth in the
model
  C_hc <- (pmin(1,w_z))*S*x_hc;# Consumption by commoners
  ##Variables defined to avoid confusion with many
equations##########################
  v.c <- 1-(C_hc/(S*x_hc));
```

```
##################################################################
##############
  # Death rate of commoners and elites incluing the concept of
famine#############
  alpha_hc <- alpha.m + ((pmax(0,v.c,na.rm = TRUE))*
                        (alpha.M - alpha.m));

##################################################################
###############

##################################################################
###########################
  employment_rate <- L/x_hc;      # Employment ratio
  Wages <- (w)*(L);               # Total wages (Real Wages* Labor)
  profit <- Y-Wages-r*D;          # Profit (Output - Total Wages - Intesrest paid on
Debt)
  profit_share <- profit/Y;       # Profit/Output
  profit_rate <- profit_share/v;  # Profit Share/v(Capital/Output)
  wage_share <- Wages/Y;          # Wages Share (Total Wages/ Output)

##################################################################
####################################
  #Simulating different forms for the Phillips Curve either it's Linear or Nonlinear
form###############
  if (type.wages==1) {
    ph_func <- (-c+d*employment_rate);
  } else if (type.wages==2) {
```

```
    ph_func <- (ph_yval - ph_min)*exp((ph_s/(ph_yval - ph_min)) *
(employment_rate - ph_xval)) + ph_min;
  }

####################################################################
######################################
  ##########################NONLINEAR INVESTMENT
CURVE######################################################
  Invest_func <- (inv_yval - inv_min)*exp((inv_s/(inv_yval -
inv_min))*(profit_rate-inv_xval)) + inv_min;
  power_input = power_factor*w_h;

####################################################################
######################################
  ####Nature Extraction as a function of Labor(L), Capital(K), Power
Input(Pi)############################
  if (type.extraction==1) {
    nature_extraction <- delta_l*y_h*L; # Nature extraction as a function of Labor
  } else if (type.extraction==2) {
    nature_extraction <- delta_k*y_h*K; # Nature extraction as a function of
Capital
  }else if(type.extraction==3){
    nature_extraction <- delta_pi*y_h*power_input;
  }

####################################################################
######################################
```

```
 ########################################STATE
EQUATIONS################################################

##################################################################
#####################################
 #With and Without Depreciation of
Wealth############################################################
###
 if (type.deprec_h==1) {
  w_h_dot <- nature_extraction - C_hc;              # Change in Wealth without
Depreciation##########
 } else if (type.deprec_h==2) {
  w_h_dot <- nature_extraction - C_hc - deprec_h*w_h; # Change in Wealth
with Depreciation#############
 }else if (type.deprec_h==3) {
  w_h_dot <- nature_extraction - C_hc - power_input;  # Change in Wealth when
Nature Extraction is a
                                             ##function of Power Input with no
Depreciation###
 }else if (type.deprec_h==4) {
  w_h_dot <- nature_extraction - C_hc - deprec_h*w_h-power_input;# Change
in Wealth when Nature
                                                    ##Extraction is a function of
 }                                                  ##Power Input with Depreciation

##################################################################
#####################################
 if (type.debt==1) {
```

```
  Investment <- Invest_func*Y;# Non linear Investment Function defined as in
Keen, 2013
 } else if (type.debt==2) {
  Investment <- profit; # Linear Investment, where investment = profit
 }
 a_dot <- alpha*a;                    # Labor productivity
 Y_dot <- ((Investment/Y/v)-gamma_deprec)*Y; # Change in Output
 x_hc_dot <- beta_hc*x_hc - alpha_hc*x_hc;  # Change in Commoner
Population
 w_dot <- ph_func*w;                  # Change in Real Wage according to the
Phillips Curve
 y_h_dot <- gamma_hc*y_h*(lambda_h - y_h) - nature_extraction;# Change in
Available Nature
 D_dot <- Investment - profit;            # Change in Debt
 list(c(a_dot,w_h_dot,Y_dot,x_hc_dot,w_dot,y_h_dot,D_dot));
}
assign("type.wages",2);    # Linear or Nonlinear Phillips Curve
assign("type.extraction",3);
assign("type.debt",1);     # Debt or No-Debt
assign("type.deprec_h",3)  # Depreciation of Wealth at a Constant Rate
assign("rtype",5)
assign("ntype",2)
assign("pftype",5)
#####################################################################
###########
#ASSIGN CONSTANT RATE OF INTERSET FROM 1% to 5% and 10%
seperately###########
```

```
######################################################################
###########
if (rtype==1) {
  assign("r",0.01);
} else if (rtype==2) {
  assign("r",0.02) ;
}else if(rtype==3){
  assign("r",0.03);
}else if(rtype==4){
  assign("r",0.04);
}else if(rtype==5){
  assign("r",0.05);
}else if(rtype==10){
  assign("r",0.1);
}
######################################################################
###########
#ASSIGN CONSTANT RATE OF INTERSET FROM 1% to 5% and 10%
seperately###########
######################################################################
###########
if (ntype==1) {
  ef <- 0.5;
} else if (ntype==2) {
  ef <-1 ;
}else if(ntype==3){
  ef <- 2.5;
}else if(ntype==4){
```

```
  ef <- 3.5
}else if(ntype==5){
  ef <- 5
}
assign("delta_l",ef*6.67e-6);   # [1/(population * yr)] depletion (production)
factor
assign("delta_k",ef*3.335e-07); # [1/(Capital * yr)] depletion (production) factor
assign("delta_pi",ef*0.3335);   # [1/(power_input* yr)] depletion (production)
factor
if (pftype==1) {                # % of wealth used as power to extract nature
  assign("power_factor",0.01);
} else if (pftype==2) {
  assign("power_factor",0.05);
}else if(pftype==3){
  assign("power_factor",0.1);
}else if(pftype==4){
  assign("power_factor",0.15);
}else if(pftype==5){
  assign("power_factor",0.2);
}

assign("c",4.8);                # Phlips Cuve = -c+d*Lambda
assign("d",5);
assign("deprec_h",0.01);  # Rate at which accumulated wealth depreciates
assign("beta_hc",0.03);   # Birth rate of the population
assign("gamma_hc",1e-2);  # Regeneration rate of nature
assign("alpha.m",0.01);   # [1/yr] normal (minimimum) death rate
assign("alpha.M",0.07);   # [1/yr] Famine (maximum) death rate
```

```
assign("S",5e-4);         # [nature/(people * yr)] subsitence salary per people
assign("rho",5e-3);       # [nature/people] subsitence salary per capita
assign("lambda_h",1*1e2); # [nature] nature carrying capacity
assign("v",3);            # accelerator (= capital/output ratio)
assign("alpha",0.02);     # rate of increase in labor productivity
assign("gamma_deprec",0.01);# depreciation rate of capital (%/yr)
######################################################################
#############
##Values for different Constants in the Nonlinear Phillips and Investment Curve
######################################################################
#############
assign("ph_xval",0.95);
assign("ph_yval",0.0);
assign("ph_s",1.6);
assign("ph_min",-0.05);
assign("inv_xval",0.03333);
assign("inv_yval",0.1);
assign("inv_s",1*2.25);
assign("inv_min",0.0);
######################################################################
##############
#SETTING INITIAL
CONDITIONS########################################################
######################################################################
##############
a_o <- 1;          # Labor Productivity
w_h_o <- 10;       # Initial Wealth
x_hc_o <- 300;     # Initial Population
```

```
y_h_o <- lambda_h;  # Initial Nature
L_o <- 0.97*x_hc_o; # Initial Labor
Y_o <- L_o*a_o;     # Initial Output (GDP)
D_o <- 0;           # Initial debt
w_o <- 0.95;        # Initial Real Wage
r_o <- r;           # Initial Constant Rate of Interest
profit_o <- Y_o - w_o*L_o - r_o*D_o;  # Initial Profits
yini <-  c(a=a_o, w_h=w_h_o,Y=Y_o,x_hc=x_hc_o,w=w_o,y_h=y_h_o,D=D_o);
times <- seq(from = 0, to = 500, by = 0.01);
out <- ode(times = times, y = yini, func = goodwinhandy, method = c("ode45"),
parms = NULL);
######################################################################
###########################
#OUTPUT FOR DIFFERENT
VARIABLES##########################################################
######
######################################################################
###########################
time <- out[,1];
labor_productivity <- out[,2];
wealth <- out[,3];
real_output <- out[,4]
population <- out[,5];
real_wages <- out[,6];
nature <- out[,7];
Debt <- out[,8];
labor <- real_output/labor_productivity;
lambda <- labor/population;
```

```
debt_ratio <- Debt/real_output;
capital <- real_output*v;
wage_share <- (real_wages/labor_productivity)*100;
employment_rate <- (labor/population)
profit_share <- (real_output - real_wages*labor - Debt*r)/real_output;
profit <- real_output - (real_wages*labor) - (r*Debt);
profit_share <- profit/real_output
profit_rate <- profit_share/v;
####################################################################
#############
# (3) Define plot arrangement and margins (in inches)
par(mfrow = c(4,2))
bmar <- 4 # Bottom margin (1)
lmar <- 5 # Left margin (2)
tmar <- 1 # Top margin (3)
rmar <- 1 # Right margin (4)
# Define plot margins and axis tick label placement
par(mar=c(bmar,lmar,tmar,rmar))
plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
plot(time,wealth,xlab = "Years",ylab =
"Wealth",type="l",cex.lab=1.5,cex.axis=1.5)
plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
```

```
plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
plot(time,wage_share,xlab="Years",ylab = "Wage
Share",type="l",cex.lab=1.5,cex.axis=1.5)
plot(time,employment_rate*100,xlab = "Years",ylab = "Employment Rate
(%)",type="l",cex.lab=1.5,cex.axis=1.5)
######################################################################
##########################################
#plot(time,capital,xlab = "years",ylab = "Capital",type = "l")
#plot(employment_rate*100,100*ph_func,xlab = "employment rate (%)",ylab =
"Annual change in real wage(%)",type = "l")#,xlim = c(95,100),ylim = c(0,20))
#plot(profit_rate*100,Invest_func*100,xlab = "profit rate(%)",ylab = "Investment
as % of Output",type = "l",xlim = c(0,10),ylim = c(0,20))
#plot(employment_rate*100,wage_share,xlab = "employment rate(%)",ylab =
"Wage share of output(%)",type = "l",cex.lab=1.5,cex.axis=1.5)
#title(main="Merged Model @ Constant r= 4.5%,L^.3K^.3a^.4", outer = TRUE,
cex =0.5, line = -1)
#plot(time,labor_productivity,xlab = "years",ylab = "labor productivity",type="l")
#plot(time,labor,xlab = "years",ylab = "labor",type="l")
#plot(time,Debt,type = "l",xlab = "years",ylab = "Debt")
#plot(time,r,xlab = "years",ylab = "rate of interest",type = "l")
#plot(employment_rate,ph_func_test,xlab = "emplotyment rate
(%)",ylab="Phillps Curve Output (change in wages)",type = "l")
#plot(time,ph_func_test,xlab = "Years",ylab="Phillps Curve Output (change in
wages)",type = "l")
#plot(profit_rate,Invest_func_test,xlab = "Profit Rate",ylab="Investment fraction
of output",type = "l")
if (ntype==1) {
```

```
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  n_DR_1 <- data.frame(debt_ratio)
  write.csv(n_DR_1,file = "Debt Ratio@n=1.csv")
  n_P_1 <- data.frame(population)
  write.csv(n_P_1,file = "Population@n=1.csv")
  n_N_1 <- data.frame(nature)
  write.csv(n_N_1,file = "Nature@n=1.csv")
  n_O_1 <- data.frame(real_output)
  write.csv(n_O_1,file = "Output@n=1.csv")
```

```
} else if (ntype==2) {
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  n_DR_2 <- data.frame(debt_ratio)
  write.csv(n_DR_2,file = "Debt Ratio@n=2.csv")
  n_P_2 <- data.frame(population)
  write.csv(n_P_2,file = "Population@n=2.csv")
  n_N_2 <- data.frame(nature)
  write.csv(n_N_2,file = "Nature@n=2.csv")
  n_O_2 <- data.frame(real_output)
```

```
  write.csv(n_O_2,file = "Output@n=2.csv")
} else if (ntype==3){
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  n_DR_3 <- data.frame(debt_ratio)
  write.csv(n_DR_3,file = "Debt Ratio@n=3.csv")
  n_P_3 <- data.frame(population)
  write.csv(n_P_3,file = "Population@n=3.csv")
  n_N_3 <- data.frame(nature)
  write.csv(n_N_3,file = "Nature@n=3.csv")
```

```
  n_O_3 <- data.frame(real_output)
  write.csv(n_O_3,file = "Output@n=3.csv")
} else if (ntype==4){
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  n_DR_4 <- data.frame(debt_ratio)
  write.csv(n_DR_4,file = "Debt Ratio@n=4.csv")
  n_P_4 <- data.frame(population)
  write.csv(n_P_4,file = "Population@n=4.csv")
  n_N_4 <- data.frame(nature)
```

```
  write.csv(n_N_4,file = "Nature@n=4.csv")
  n_O_4 <- data.frame(real_output)
  write.csv(n_O_4,file = "Output@n=4.csv")
} else if(ntype==5){
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  n_DR_5 <- data.frame(debt_ratio)
  write.csv(n_DR_5,file = "Debt Ratio@n=5.csv")
  n_P_5 <- data.frame(population)
  write.csv(n_P_5,file = "Population@n=5.csv")
```

```
  n_N_5 <- data.frame(nature)
  write.csv(n_N_5,file = "Nature@n=5.csv")
  n_O_5 <- data.frame(real_output)
  write.csv(n_O_5,file = "Output@n=5.csv")
}
######################################################################
################
if (rtype==1) {
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
```

```
  r_DR_1 <- data.frame(debt_ratio)
  write.csv(r_DR_1,file = "Debt Ratio@r=1%.csv")
  r_P_1 <- data.frame(population)
  write.csv(r_P_1,file = "Population@r=1%.csv")
  r_N_1 <- data.frame(nature)
  write.csv(r_N_1,file = "Nature@r=1%.csv")
  r_O_1 <- data.frame(real_output)
  write.csv(r_O_1,file = "Output@r=1%.csv")
} else if (rtype==2) {
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
```

```
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  r_DR_2 <- data.frame(debt_ratio)
  write.csv(r_DR_2,file = "Debt Ratio@r=2%.csv")
  r_P_2 <- data.frame(population)
  write.csv(r_P_2,file = "Population@r=2%.csv")
  r_N_2 <- data.frame(nature)
  write.csv(r_N_2,file = "Nature@r=2%.csv")
  r_O_2 <- data.frame(real_output)
  write.csv(r_O_2,file = "Output@r=2%.csv")
} else if (rtype==3){
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
```

```
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  r_DR_3 <- data.frame(debt_ratio)
  write.csv(r_DR_3,file = "Debt Ratio@r=3%.csv")
  r_P_3 <- data.frame(population)
  write.csv(r_P_3,file = "Population@r=3%.csv")
  r_N_3 <- data.frame(nature)
  write.csv(r_N_3,file = "Nature@r=3%.csv")
  r_O_3 <- data.frame(real_output)
  write.csv(r_O_3,file = "Output@r=3%.csv")
} else if (rtype==4){
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
```

```
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  r_DR_4 <- data.frame(debt_ratio)
  write.csv(r_DR_4,file = "Debt Ratio@r=4%.csv")
  r_P_4 <- data.frame(population)
  write.csv(r_P_4,file = "Population@r=4%.csv")
  r_N_4 <- data.frame(nature)
  write.csv(r_N_4,file = "Nature@r=4%.csv")
  r_O_4 <- data.frame(real_output)
  write.csv(r_O_4,file = "Output@r=4%.csv")
} else if(rtype==5){
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
```

```
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  r_DR_5 <- data.frame(debt_ratio)
  write.csv(r_DR_5,file = "Debt Ratio@r=5%.csv")
  r_P_5 <- data.frame(population)
  write.csv(r_P_5,file = "Population@r=5%.csv")
  r_N_5 <- data.frame(nature)
  write.csv(r_N_5,file = "Nature@r=5%.csv")
  r_O_5 <- data.frame(real_output)
  write.csv(r_O_5,file = "Output@r=5%.csv")
}else if (rtype==10) {
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
```

```
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  r_DR_10 <- data.frame(debt_ratio)
  write.csv(r_DR_10,file = "Debt Ratio@r=10%.csv")
  r_P_10 <- data.frame(population)
  write.csv(r_P_10,file = "Population@r=10%.csv")
  r_N_10 <- data.frame(nature)
  write.csv(r_N_10,file = "Nature@r=10%.csv")
  r_O_10 <- data.frame(real_output)
  write.csv(r_O_10,file = "Output@r=10%.csv")
}
if (pftype==1) {
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
```

```
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  pf_DR_1 <- data.frame(debt_ratio)
  write.csv(pf_DR_1,file = "Debt Ratio@pf=1.csv")
  pf_P_1 <- data.frame(population)
  write.csv(pf_P_1,file = "Population@pf=1.csv")
  pf_N_1 <- data.frame(nature)
  write.csv(pf_N_1,file = "Nature@pf=1.csv")
  pf_O_1 <- data.frame(real_output)
  write.csv(pf_O_1,file = "Output@pf=1.csv")
} else if (pftype==2) {
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
```

```
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  pf_DR_2 <- data.frame(debt_ratio)
  write.csv(pf_DR_2,file = "Debt Ratio@pf=2.csv")
  pf_P_2 <- data.frame(population)
  write.csv(pf_P_2,file = "Population@pf=2.csv")
  pf_N_2 <- data.frame(nature)
  write.csv(pf_N_2,file = "Nature@pf=2.csv")
  pf_O_2 <- data.frame(real_output)
  write.csv(pf_O_2,file = "Output@pf=2.csv")
} else if (pftype==3){
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)
```

```
  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  pf_DR_3 <- data.frame(debt_ratio)
  write.csv(pf_DR_3,file = "Debt Ratio@pf=3.csv")
  pf_P_3 <- data.frame(population)
  write.csv(pf_P_3,file = "Population@pf=3.csv")
  pf_N_3 <- data.frame(nature)
  write.csv(pf_N_3,file = "Nature@pf=3.csv")
  pf_O_3 <- data.frame(real_output)
  write.csv(pf_O_3,file = "Output@pf=3.csv")
} else if (pftype==4){
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
  rmar <- 1 # Right margin (4)
```

```
  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  pf_DR_4 <- data.frame(debt_ratio)
  write.csv(pf_DR_4,file = "Debt Ratio@pf=4.csv")
  pf_P_4 <- data.frame(population)
  write.csv(pf_P_4,file = "Population@pf=4.csv")
  pf_N_4 <- data.frame(nature)
  write.csv(pf_N_4,file = "Nature@pf=4.csv")
  pf_O_4 <- data.frame(real_output)
  write.csv(pf_O_4,file = "Output@pf=4.csv")
} else if(pftype==5)
  par(mfrow = c(3,2))
  bmar <- 4 # Bottom margin (1)
  lmar <- 5 # Left margin (2)
  tmar <- 2 # Top margin (3)
```

```
  rmar <- 1 # Right margin (4)

  # Define plot margins and axis tick label placement
  par(mar=c(bmar,lmar,tmar,rmar))
  plot(time,wealth,xlab = "Years",ylab = "Wealth
Accumulated",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,population,xlab = "Years",ylab =
"Population",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,nature,xlab = "Years",ylab = "Nature",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,real_output,xlab = "Years",ylab = "Real
Output",type="l",cex.lab=1.5,cex.axis=1.5)
  plot(time,debt_ratio,xlab = "Years",ylab="Debt Ratio",type =
"l",cex.lab=1.5,cex.axis=1.5)
  plot(time,profit_rate,xlab = "Years",ylab = "Profit
Rate",type="l",cex.lab=1.5,cex.axis=1.5)
  pf_DR_5 <- data.frame(debt_ratio)
  write.csv(pf_DR_5,file = "Debt Ratio@pf=5.csv")
  pf_P_5 <- data.frame(population)
  write.csv(pf_P_5,file = "Population@pf=5.csv")
  pf_N_5 <- data.frame(nature)
  write.csv(pf_N_5,file = "Nature@pf=5.csv")
  pf_O_5 <- data.frame(real_output)
  write.csv(pf_O_5,file = "Output@pf=5.csv")
```

8.3.1. CODES TO READ .CSV OUTPUT FILES FROM THE PREVIOUS CODE

#This code reads the .CSV output files and combines them to create plots to run sensitivity

```
##analysis on Debt Ratio, Population, Nature, and Output.
DR1 <- read.csv("Debt Ratio@pf=1.csv")
DR2 <- read.csv("Debt Ratio@pf=2.csv")
DR3 <- read.csv("Debt Ratio@pf=3.csv")
DR4 <- read.csv("Debt Ratio@pf=4.csv")
DR5 <- read.csv("Debt Ratio@pf=5.csv")
DRCombined <- cbind(DR1,DR2,DR3,DR4,DR5)
DR_C <- data.frame(DRCombined)
write.csv(DR_C,file = "DebtRatioCombined.csv")
#####################################################
p1 <- read.csv("Population@pf=1.csv")
p2 <- read.csv("Population@pf=2.csv")
p3 <- read.csv("Population@pf=3.csv")
p4 <- read.csv("Population@pf=4.csv")
p5 <- read.csv("Population@pf=5.csv")
pCombined <- cbind(p1,p2,p3,p4,p5)
p_C <- data.frame(pCombined)
write.csv(p_C,file = "populationCombined.csv")
#####################################################
n1 <- read.csv("Nature@pf=1.csv")
n2 <- read.csv("Nature@pf=2.csv")
n3 <- read.csv("Nature@pf=3.csv")
n4 <- read.csv("Nature@pf=4.csv")
n5 <- read.csv("Nature@pf=5.csv")
nCombined <- cbind(n1,n2,n3,n4,n5)
n_C <- data.frame(nCombined)
write.csv(n_C,file = "natureCombined.csv")
######################################################
```

```
o1 <- read.csv("Output@pf=1.csv")
o2 <- read.csv("Output@pf=2.csv")
o3 <- read.csv("Output@pf=3.csv")
o4 <- read.csv("Output@pf=4.csv")
o5 <- read.csv("Output@pf=5.csv")
oCombined <- cbind(o1,o2,o3,o4,o5)
o_C <- data.frame(oCombined)
write.csv(o_C,file = "outputCombined.csv")
```

8.3.2. PLOTTING CODES FOR THE MERGED CSV'S

```
#This code plots the CSV outputs from the previous code, it's important to make
sure that
##the CSV's do not contain the time column more than once before running the
code.
debtratio <- read.csv("DebtRatioCombined.csv")
population <- read.csv("populationCombined.csv")
Nature <- read.csv("natureCombined.csv")
Output <- read.csv("outputCombined.csv")
par(mfrow = c(2,2))
bmar <- 4 # Bottom margin (1)
lmar <- 5 # Left margin (2)
tmar <- 2 # Top margin (3)
rmar <- 1 # Right margin (4)
#
# # # Define plot margins and axis tick label placement
par(mar=c(bmar,lmar,tmar,rmar))
```

```
matplot(debtratio[,1]/100,debtratio[,-1],type = "l",lty=1,lwd =2,xlab =
"Years",ylab = "Debt Ratio",col = c("black","blue","red","green","brown"),ylim =
c(0,5e4),xlim = c(0,500),cex.lab=1.5,cex.axis=1.5)
legend(150,5e4,legend=c("1%", "5%","10%","15%","20%"),lty=1, cex=0.8,col =
c("black","blue","red","green","brown"))
matplot(population[,1]/100,population[,-1],type = "l",lty=1,lwd =2,xlab =
"Years",ylab = "Population",col =
c("black","blue","red","green","brown"),cex.lab=1.5,cex.axis=1.5)#,ylim =
c(0,8e4))#,xlim = c(0,300))
legend(0,5.5e4,legend=c("1%", "5%","10%","15%","20%"),lty=1, cex=0.8,col =
c("black","blue","red","green","brown"))
matplot(Nature[,1]/100,Nature[,-1],type = "l",lty=1,lwd =2,xlab = "Years",ylab =
"Available Nature",col =
c("black","blue","red","green","brown"),cex.lab=1.5,cex.axis=1.5)#,xlim =
c(0,300))
legend(100,90,legend=c("1%", "5%","10%","15%","20%"),lty=1, cex=0.8,col =
c("black","blue","red","green","brown"))
matplot(Output[,1]/100,Output[,-1],type = "l",lty=1,lwd =2,xlab = "Years",ylab =
"Output",col = c("black","blue","red","green","brown"),ylim =
c(0,2e8),cex.lab=1.5,cex.axis=1.5)#,xlim = c(0,300))
legend(0,2e8,legend=c("1%", "5%","10%","15%","20%"),lty=1, cex=0.8,col =
c("black","blue","red","green","brown"))
title(main = "Effect of Constant Rate of Interest", outer = TRUE, cex =2.5, line =
-1  )
#legend(10,5,lty=c(1,1),lwd=c(2.5,2.5))
#legend(0,1e09,legend=c("Line 1", "Line 2"),lty=1:2, cex=0.8)
```

9. REFERENCE

Acemoglu, Daron. 2009. "The Solow Growth Model." Introduction to Modern Economic Growth 26-76.

Antón, Susan C., and Carl C. Swisher III. 2005. "Early Dispersals of homo from Africa." Annual Review of Anthropology 271-296.

Arrow, K. J., H. B. Chenery, B. S. Minhas, and R. M. Solow. 1961. "Capital-labor substitution and economic efficiency." Review of Economics and Statistics (The MIT Press) 225-250.

Chen, J. 2016. The Unity of Science and Economics. New York: Springer-Verlag.

Cobb, C. W., and P. H. Douglas. 1928. "A Theory of Production." American Economic Review 139-165.

Dixon, H. 1999. "New Keynesian Economics, nominal rigidities and involuntary unemployment." Journal of Economic Methodology 221-238.

Dodson. 2004. 46.

Domar, Evsey. 1946. "Capital Expansion, Rate of Growth, and Employment." Econometrica.14(2) 137-147.

Douglas, Paul H. 1976. "The Cobb-Douglas Production Function Once Again: Its History, Its Testing, and Some New Empirical Values." Journal Political Economy 84.

Foley, K. D., and T. Michl. 1999. Growth and Distribution. Cambridge, MA: Harvard University Press.

Godley, Wynne, and Marc Lavoie. 2006. Monetary Economics: "An Integrated Approach to Credit, Money, Income, Production and Wealth. palgrave Macmillan.

Goodwin, R.M. 1967. A Growth Cycle. Edited by C.H. Feinstein. Cambridge: Cambridge University Press.

Harrod, Roy F. 1939. "An Essay in Dynamic Theory." The Economic Journal. 49 (193) 14-33.

Jorgensen, Dale W. 2000. Econometrics, vol 1: Econometric Modelling of Producer Behavior. Cambridge: The MIT Press.

Keen, S. 2013. "A Monetary Minsky Model of the Great Moderation and the Great Recession." Journal of Economic Behavior & Organization 221-235.

Keen, S. 2013a. "A Monetary Minsky Model of the Great Recession." Journal of Economic Behavior & Organization, 86 221-235.

Keen, S. 1995. "Finance and Economic Breakdown: Modeling Minsky's "Financial Instability Hypothesis" ." Journal of Economic Behavior & Organization, 86 221-235.

Keen, S. 2009. "Neoclassical Economics: mad, bad, and dangerous to know." Steve Keen's Debtwatch.

Keynes, John Maynard. 1936. The general theory of employment, interest, and money. London: Macmillan.

King, C. W. 2015b. "The Rising Cost of Resources and Global Indicators of Change." American Scientist 103.

Klump, R., A. William, and P. McAdam. 2007. "Factor Substitution and Factor Augmenting Technical Progress in the US: A Normalized Supply-Side Approach." Review of Economics and Statistics 183-192.

Lifshitz, Michail. 1948. "Giambattista Vico (1668-1744)." Philosophy and Phenomenological Research 8.3 391-414.

Lotka, Alfred J. 1925. Elements of Physical Biology. Williams and Wilkins.

Malmström, Vincent H. 2014. "The Maya Inheritance."

Meadows, Donella H., Dennis L. Meadows, Jorgen Randers, and Willian W. Behrens III. 1972. The Limits to Growth. 1972 Universe Books.

Mitchell, T. 2013. Carbon Democracy: Political Power in the Age of Oil. London and New York: Verso.

Morgan, David. 2007. The Mongols, 2nd Edition. Wiley-Blackwell.

Morgan, Mary S. 2008. "models." The New Palgrave Dictionary of Economics, 2nd Edition.

Morris, Ian. 2010. Why the West Rules - For Now: The Patterns of History, and What They Reveal About the Future. New York: Farrar, Straus, and Giroux.

Motesharrei, S., J. Rivas, and E. Kalnay. 2014. "Human and Nature Dynamics (HANDY): Modeling inequality and use of resources in the collapse or sustainability of societies." Ecological Economics, 101 90-102.

Phillips, A. W. 1958. "The Relationship between Unemployment and the Rate of Change of Money Wages in the United Kingdon 1861-1957." Economica.25(100) 283-299.

Pindyck, R.S. 2013. "Climate Change Policy: What Do the Models Tell Us?" Journal of Economic Literature, 51 860-872.

Samuelson, P.A. 1939. "Interactions Between the Multiplier Analysis and the Principle of Acceleration." Review of Economic Statistics 75-78.

Solow, Robert M. 1956. "A contribution to the theory of economic growth." Quarterly Journal of Economics. Oxford Journals. 70(1) 65-94.

Stanford, James. 1993. "Continental Economic Integration: Modeling the Impact on Labor." Annals of the American Academy of Political and Social Science 92-110.

Stern, N. 2013. "The Structure of Economic Modeling of the Potential Impacts of Climate Change: Grafting Gross Underestimation of Risk onto Already Narrow Science Models." Journal of Economic Literature, 51 838-859.

Swan, Trevor W. 1956. "Economic growth and capital accumulation." Economic Record 334-361.

Tainter, J. A., T. F. H. Allen, A. Little, and T. W. Hoekstra. 2003. "Resource Transitions and Energy Gain: Contexts of Organization." Conservation Ecology,7.

Tainter, Joseph A. 2011. "Energy, Complexity, and Sustainability: A Historical Perspective." Environmental Innovation and Societal Transitions (Cambridge University Press) 89-195.

Tainter, Joseph A. 1988. The Collapse of Complex Societies. Cambridge University Press.

Trinkaus, Erik. 2005. "Early Modern Humans." Annual Review of Anthropology 207-230.

Turchin, P., and S. A. Nefedov. 2009. Secular Cycles. Princeton University Press.

Turchin, P., T.E. Currie, E.A.L. Turner, and S. Gavrilets. 2013. "War, space, and the evolution of Old Worlds complex societies." Proceedings of the National Academy of Sciences 110.

Wright, Rita P. 2009. "The Ancient Indus: Urbanism, Economy, and Society."